D3AMT2202 **(2 Pages)** **Name..............................**

Reg.No..........................

THIRD SEMESTER M.Sc. DEGREE EXAMINATION, NOVEMBER 2023
(Regular/Improvement/Supplementary)
MATHEMATICS
FMTH3C12 - COMPLEX ANALYSIS

Time: 3 Hours **Maximum Weightage: 30**

Part A: Answer *all* questions. Each carries *1* weightage.

1. Show that the series $\sum_{n=1}^{\infty} \frac{(-1)^n}{n} z^{n(n+1)}$ has radius of convergence 1.
2. Give the principal branch of $\sqrt{1-z}$.
3. Evaluate the cross ratio of $(i-1, \infty, 1+i, 0)$
4. Let $r(t) = 1 + e^{it}$, for $0 \leq t \leq 2\pi$. Find $\int_r (z^2 - 1)^{-1} dz$.
5. Prove that bounded entire function is a constant.
6. Let f and g be two analytic functions on a region G such that $f(z)g(z) = 0$ for all z in G. Prove that $f \equiv 0$ or $g \equiv 0$.
7. Give Laurent expansion of $f(z) = \frac{1}{z(z-1)(z-2)}$ on $0 < |z| < 1$.
8. Suppose f has a simple pole at $z = a$ and let g be analytic in an open set containing a Prove that $\text{Res}(fg; a) = g(a)\text{Res}(f; a)$.

(8x1= 8 weightage)

Part B: Answer *any two* questions *from each unit.* Each carries *2* weightage.

Unit *I*

9. Prove that a Mobius transformation takes circles onto circles.
10. If f is a branch of the logarithm in a region G, show that f is analytic in G and its derivative is $1/z$.
11. Suppose $f : G \longrightarrow \mathbb{C}$ is analytic and G is connected. Show that if $f(z)$ is real for all z in G then f is constant.

Unit *II*

12. State and prove fundamental theorem of algebra.

(P.T.O.)

13. Let f be a one one, analytic function on a region G. Show that $f'(z) \neq 0$ for any z in G.

14. Let f and g be analytic in a region G. Prove that $f \equiv g$ if and only if $\{z \in G : f(z) = g(z)\}$ has a limit point in G.

Unit III

15. Show that $\int_0^\infty \frac{\sin x}{x} dx = \frac{\pi}{2}$.

16. Prove that an entire function has a pole at infinity of order m if and only if it is a polynomial of degree m.

17. State and prove Rouche's theorem.

(6x2= 12 weightage)

Part C: Answer *any two* questions. Each carries *5* weightage.

18. If G is simply connected and $f : G \longrightarrow \mathbb{C}$ is analytic in G, then prove that f has a primitive in G.

19. If γ_0 and γ_1 are two closed rectifiable curves in G, and $\gamma_0 \sim \gamma_1$, then prove that $\int_{\gamma_0} f = \int_{\gamma_1} f$ for every analytic function f in G.

20. State and prove the theorem on the Laurent series development in the annulus.

21. State and prove Residue theorem.

(2x5= 10 weightage)