D3AST2102 (3 Pages) Name.....
Reg.No.....

THIRD SEMESTER M.Sc. DEGREE EXAMINATION, NOVEMBER 2022
(Regular/Improvement/Supplementary)
STATISTICS
FMST3C12: TESTING OF STATISTICAL HYPOTHESES

Time: 3 Hours **Maximum Weightage: 30**

Part A: Answer any four questions.
Each carries 2 weightage.

1. **Outline the methodology of a sequential probability ratio test.**
2. **Define a UMP test. Will it always exist? Justify your answer with the help of an example.**
3. **Define type I error, type II error, size and power of test.**
4. **Define $\alpha-$similar tests. Explain tests with Neyman structure.**
5. **Narrate various advantages and disadvantages of using non-parametric tests.**
6. **Explain chi-square test for goodness of fit.**
7. **Explain: (a) locally most powerful test (b) locally most powerful unbiased test.**

(4 x 2 = 8 weightage)

Part B: Answer any four questions.
Each carries 3 weightage.

8. **Let $X \sim N(\mu, 4)$; μ is unknown. To test $H_0 : \mu = -1$ against $H_1 : \mu = 1$ based on a sample of size 10 from this population, we use the critical region $x_1 + 2\,x_2 + ... + 10\,x_{10} \geq 0$. What is its size? What is the power of the test?**
9. **Show that each of the following families has an MLR.**
   (a) $N(\theta, \sigma^2)$ family with σ^2 known.
   (b) $P(\lambda)$ family

10. **Compare Chi-square test of goodness of fit with Kolmogorov-Smirnov test.**

11. **A sample of size 1 is taken from the pdf**

$$f_\theta(x) = \frac{2}{\theta^2}(\theta - x), \quad 0 < x < \theta$$

Find an MP test of $H_0 : \theta = \theta_0$ against $H_1 : \theta_1$ $(\theta_1 < \theta_0)$.

12. **Define OC function and ASN function of SPRT. Show that SPRT terminates with probability one under certain assumptions.**

13. **How Wilcoxon's signed-rank test differ from sign test and how to perform it?**

14. **Explain general method of construction of likelihood ratio test. Obtain likelihood ratio test for testing $H_0 : \mu = \mu_0$ against $H_1 : \mu = \mu_1$ where $X_i \sim N(\mu, \sigma^2)$, σ^2 unknown.**

(4 x 3= 12 weightage)

Part C: Answer any two questions.
Each carries 5 weightage.

15. **a) Let $x_1, x_2, ..., x_n$ be random sample from $U(0, \theta)$. Obtain MP test for testing $H_0 : \theta = 1$ against $H_1 : \theta = 2$ with $\alpha = 0.05$.**
b) Show that Cauchy distribution does not have MLR property.

16. **Using Neyman-Pearson lemma, find the best critical region for the test of hypothesis $H_0 : \mu = \mu_0$ vs $H_1 : \mu = \mu_1$ for the $N(\mu, \sigma^2)$ population, when σ^2 is known in the cases**
(i) $\mu_0 < \mu_1$ (ii) $\mu_0 > \mu_1$
Also find the power of the test.

17. **State and prove the properties of SPRT.**

18. **Point out the advantages of SPRT over classical tests. Find OC function of the SPRT for testing $H_0 : \theta = 0.2$ against $H_1 : \theta = 0.4$ where is the probability of success of Bernoulli trial. Take $\alpha = \beta = 0.05$.**

(2 x 5 = 10 weightage)