D3AST2101 **(2 Pages)** **Name.............................**
Reg.No..........................

THIRD SEMESTER M.Sc. DEGREE EXAMINATION, NOVEMBER 2022
(Regular/Improvement/Supplementary)

STATISTICS
FMST3C11 - STOCHASTIC PROCESSES

Time: 3 Hours **Maximum Weightage: 30**

Part A: Answer any *four* questions. Each carries *two* weightage.

1. Define a stochastic process and state the classifications of stochastic process with examples.
2. Define a discrete time Markov chain. Derive the Chapman-Kolmogorov equation for such process.
3. Define a Poisson process. Obtain the probability distribution of the number of events occurred in an interval (0, t].
4. Define a compound Poisson process. Obtain its mean and variance.
5. Define (a) renewal process and (b) renewal function.
6. Describe an M/M/1 queue.
7. Describe a multiserver queueing model.

(4 × 2 = 8 weightage)

Part B: Answer any *four* questions. Each carries *three* weightage.

8. Define recurrent Markov chain. Examine whether one dimensional symmetric random walk on $\{0, \pm 1, \pm 2, ...\}$ is recurrent.
9. Describe Gambler's ruin problem. Find the probability of ultimate ruin of the Gambler.
10. Find the distribution of the waiting time for the occurrence of the n^{th} event in a Poisson process, with parameter λ.
11. (a) Show that the interarrival time of a Poisson process is exponentially distributed. (b) Define a birth and death process $\{X(t),\ t \geq 0\}$ with birth rate $\{\lambda_n\}$ and death rate $\{\mu_n\}$. Derive an expression for $P(X(t) = n)$ under the steady state.
12. (a) If the mean function of a renewal process $\{N(t)\}$ is $m(t) = \frac{t}{2}$, what is $P(N(5) = 0)$?
    (b) State and prove the elementary renewal theorem.
13. (a) Compute the renewal function when the interarrival distribution F is such that

    $$1 - F(t) = pe^{-\mu_1 t} + (1 - p)e^{-\mu_2 t}.$$

(P.T.O.)

(b) Is the sum of two independent renewal processes a renewal process? Establish your claim.

14. Describe the queueing problem and explain the basic characteristics of a queue.

(4 × 3 = 12 weightage)

Part C: Answer any *two* questions. Each carries *five* weightage.

15. Describe Galton Watson branching process $\{X_n\}$, with offspring distribution having probability generating function (pgf)$\emptyset(s)$. Show that in the usual notations

$\emptyset_{n+1}(s) = \emptyset_n\big(\emptyset(s)\big)$. Also find $E(X_{n)}$ and $Var(X_n)$.

16. (a) Define a Brownian motion with drift and geometric Brownian motion.

(b) Obtain the integral equation satisfied by the renewal function.

17. (a) Describe: (i) Renewal reward process and (ii) Regenerative process.

(b) If $\{N(t), t \geq 0\}$ is a renewal process generated by the distribution function F show that E(N(t)) and F determine each other uniquely.

18. For an M/M/1 queueing system with FCFS queue discipline under the steady state, derive the distributions for waiting time in the queue and waiting time in the system.

(2 × 5 = 10 weightage)