D3AMT2105 (2 Pages) Name..............................
Reg.No..........................

THIRD SEMESTER M.Sc. DEGREE EXAMINATION, NOVEMBER 2022
MATHEMATICS
FMTH3E01: CODING THEORY

Time: 3 Hours **Maximum Weightage: 30**

PART A: Answer ALL questions. Each carries 1 weightage.

1. Compute the weight of each of the following words and the distance between each pair of them: $u = 1001010$, $v = 0110101$, $w = 0011110$ and $s = u + v + w$.
2. Define a linear code. Is $C = \{0000, 0101, 1010, 1100\}$ a linear code?
3. Define an algorithm for finding a basis for the dual code $C^{\perp}$
4. Define a perfect code and give an example of a perfect code.
5. Describe the r^{th} order Reed Muller code.
6. Does there exists a $(15, 7, 5)$ linear code?
7. Define cyclic shift and cyclic code.
8. Find the generator polynomial of the dual of the cyclic code C of length $n = 7$ and having generator polynomial $g(x) = 1 + x + x^3$.

PART B: Answer ANY TWO questions FROM EACH UNIT. Each carries 2 weightage.

UNIT I

9. If $C = \{01000, 01001, 00011, 11001\}$ and a word $w = 10110$ is received, which codeword was most likely to have been sent?
10. Show that a code C of distance d will correct all error patterns of weight less than or equal to $\lfloor \frac{d-1}{2} \rfloor$ and there is at least one error pattern of weight $1 + \lfloor \frac{d-1}{2} \rfloor$ which C will not correct.
11. For $S = \{1010, 0101, 1111\}$ and $C =< S >$, compute the dual code $C^{\perp}$.

UNIT II

12. Prove that $RM(r-1, m)$ is contained in $RM(r, m)$, $r > 0$.
13. Show that an extended Hamming code of length 8 is a self-dual code.
14. Prove the following.
    (a) Any perfect code of length and distance $2t + 1$ has exactly 2 code words.
    (b) Hamming codes are perfect single-error correcting codes.

(P.T.O.)

UNIT III

15. **Find a parity check matrix for the cyclic code of length 7 having generator polynomial** $g(x) = 1 + x + x^2 + x^4$**.**

16. **If** C **is a linear cyclic code of length** n **and dimension** k **with generator** $g(x)$ **and if** $1 + x^n = g(x)h(x)$**, then prove that** $C^{\perp}$ **is a cyclic code of dimension** $n - k$ **with generator** $x^k h(x^{-1})$

17. **Let** $\alpha \neq 0$ **be an element of** $GF(2^r)$**. Let** $m_\alpha(x)$ **be the minimal polynomial of** α**. Then prove the following.**

- **(a)** $m_\alpha(x)$ **is irreducible over** K**.**
- **(b) If** $f(x)$ **is any polynomial over** K **such that** $f(\alpha) = 0$**, then** $m_\alpha(x)$ **is a factor of** $f(x)$**.**
- **(c) The minimal polynomial is unique**
- **(d) The minimal polynomial** $m_\alpha(x)$ **is a factor of** $1 + x^{2^r - 1}$**.**

PART C: Answer ANY TWO questions. Each carries 5 weightage.

18. **(a) Using the elementary row operation of a matrix, find a basis for** $C = < S >$**, where** $S = \{10101, 01010, 11111, 00011, 10110\}$**.**

(b) Given is a parity-check matrix for a linear code C**.**

$$H = \begin{pmatrix} .1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

. Find generator matrices for C **and** $C^{\perp}$**.**

19. **Prove the following.**

- **(a) The extended Golay code** C_{24} **is self dual.**
- **(b) The distance of** C_{24} **is 8.**
- **(c)** C_{24} **is a three error-correcting code.**

20. **Find the minimal polynomial of each element of** $GF(2^4)$ **constructed using** $p(x) = 1 + x^3 + x^4$**.**

21. **Let** $g(x) = 1 + x^4 + x^6 + x^7 + x^8$ **be the generator polynomial for a cyclic code of length** $n = 15$ **and** $d = 5$**. Decode the received word** $w = 110011100111000$.