D3AMT2104 (2 Pages) Name.............................
Reg.No...........................

THIRD SEMESTER M.Sc. DEGREE EXAMINATION, NOVEMBER 2022
(Regular/Improvement/Supplementary)
MATHEMATICS
FMTH3C14: PDE AND INTEGRAL EQUATIONS

Time: 3 Hours **Maximum Weightage: 30**

Part A

Answer all questions. Each carries 1 weightage

1. Define first order linear partial differential eauation and a quasi linear partial differential equation. Give one example for each.
2. Explain the method of characteristics to find solution of a first order quasi linear partial differential equation.
3. Explain the terms "Strip Equations" and "Characteristic Strips" associated to a general first order partial differential equation.
4. Explain the classification of second order linear partial differential equations. Give example of a parabolic type equation.
5. Write the one dimensional heat equation and specify the initial conditions.
6. Explain the terms "domain of dependence" and "range of influence".
7. Write the different types of integral equations and give one example for each.
8. Define separable kernel of an integral equation. Give one example of such a kernel.

(8×1= 8 weightage)

Part B

Answer any two questions from each unit. Each carries 2 weightage.

Unit I

9. Solve the equation $u_x = 1$ subject to the initial condition $u(0, y) = g(y)$ by the method of characteristics.
10. Solve the equation $(y + u)u_x + yu_y = x - y$
subject to the initial condition $u(x, 1) = 1 + x$.
11. Prove that the equation $u_{xx} + 6u_{xy} - 16u_{yy} = 0$ is hyperbolic.
Find its canonical form and the general solution.

Unit II

12. State and prove the weak maximum principle.

13. State and prove the mean value principle of a harmonic function in a planar domain.

14. Let $u(x, y)$ be the harmonic function in the unit square satisfying the Dirichlet conditions $u(x, 0) = 1 + \sin \pi x$, $u(x, 1) = 2$, $u(0, y) = u(1, y) = 1 + y$.
Represent u as a sum of a harmonic polynomial, and a harmonic function $v(x, y)$ that satisfies the compatibility condition of Dirichlet's problem on a rectangle.

Unit III

15. If $I_n(x) = \int_a^x (x - \xi))^{n-1} f(\xi) d\xi$ were n is a positive integer and a is a constant then prove that $\dfrac{d^n I_n}{dx^n} = (n-1)! f(x)$.

16. Transform the initial value problem $\dfrac{d^2y}{dx^2} + \lambda y = f(x), \quad y(0) = 1, \quad y'(0) = 0$ into an integral equation.

17. Prove that the characteristic numbers of a Fredholm Integral Equation with a real symmetric kernel are all real.

(6×2= 12 weightage)

Section C

Answer any two questions. Each carries 5 weightage

18. Solve the equation $u_x + 3y^{2/3} u_y = 2$,
subject to the initial condition $u(x, 1) = 1 + x$.

19. (a) Explain the d'Alembert's method of solution of a Cauchy's problem for the one-dimensional homogeneous wave equation.
    (b) Solve the Cauchy problem $u_{tt} - u_{xx} = 0, \quad -\infty < x < \infty, \quad t > 0$

$$u(x,0) = f(x) \begin{cases} 0 & \text{if } -\infty < x < -1 \\ x+1 & \text{if } -1 \le x \le 0 \\ 1-x & \text{if } 0 \le x \le 1 \\ 0 & \text{if } 1 \le x < \infty \end{cases} \qquad u_t(x,0) = g(x) \begin{cases} 0 & \text{if } -\infty < x < -1 \\ 1 & \text{if } -1 \le x \le 1 \\ 0 & \text{if } 1 \le x < \infty \end{cases}$$

20. Describe the prolem of a vibrating string without external forces and with two clamped but free ends. Also describe the method of separation of variables to solve the problem.

21. Solve the integral equation $y(x) = \lambda \int_0^1 (1 - 3x\xi) y(\xi) d\xi + F(x)$ using iteration method.

(2×5= 10 weightage)