D3AMT2102 **(2 Pages)** **Name.............................**
Reg.No..........................

THIRD SEMESTER M. Sc. DEGREE EXAMINATION, NOVEMBER 2022
(Regular/Improvement/Supplementary)

MATHEMATICS
FMTH3C12- COMPLEX ANALYSIS

Time: 3 Hours **Maximum Weightage: 30**

Part A: Answer *all* questions. Each carries 1 weightage.

1. For each of the following complex numbers give the corresponding points of the Riemann sphere. 0, 1+i, 3+2i.
2. By an example show that a piecewise smooth path need not be a smooth path.
3. Give an example of a complex function which is nowhere analytic. Substantiate your claim.
4. Define the line integral $\int_{\gamma} f(z)dz$ of a function $f(z)$ along a rectifiable path γ. Further show that $\int_{-\gamma} f(z)dz = -\int_{\gamma} f(z)dz$.
5. Define zero of an analytic function and prove that zeroes are isolated.
6. State the first version and homotopic version of Cauchy's theorem.
7. Find the singularities if any and specify its nature for the function $e^{\frac{1}{z-2+i}}$.
8. Using Cauchy's residue theorem evaluate $\int_{\gamma} \frac{-3z}{(2z-3)}dz$ where γ is the positively oriented circle $|z| = \frac{5}{2}$.

(8 × 1 = 8 weightage)

Part B: Answer any *two* questions from each unit. Each carries 2 weightage.

Unit 1

9. Find the subset of the Riemann sphere S which correspond to the real and imaginary axes of the complex plane $\mathbf{C}$.
10. Show that a power series represents an analytic function inside its circle of convergence.
11. Define the cross-ratio of four points and show that it is invariant under a Mobius transformation.

(P.T.O.)

Unit 2

12. Prove that an analytic function is infinitely differentiable and find a formula for its n^{th} derivative.

13. State and prove open mapping theorem.

14. State and prove the first version of Cauchy's integral formula.

Unit 3

15. Find the Laurent's series expansion of $\dfrac{1}{z(z-1)(z-2)}$ about its singularities.

16. State and prove Cauchy's Residue Theorem.

17. Using Rouche's find the number of roots of the equation $z^4 - 6z + 3 = 0$ having their moduli between 1 and 2.

(6 × 2 = 12 weightage)

Part C: Answer any *two* questions. Each carries 5 weightage.

18. a) Derive a set of necessary and sufficient conditions for $f(z) = u + iv$ to be analytic at a point.

    b) Show that cross-ratio of four points is real if and only if the four points lie on a circle.

19. a) State and prove maximum modulus principle.

    b) Show that a non-constant analytic function maps open sets into open sets.

20. a) If f has an isolated singularity at a prove that $z = a$ is a removable singularity if and only if $\lim_{z \to a}(z - a)f(z) = 0$.

    b) State and prove Casorati-Weierstrass theorem on essential singularities.

21. a) State and prove the argument principle.

    b) Using Residue Theorem, choosing a suitable contour, show that $\int_0^\infty \frac{\sin x}{x}dx = \frac{\pi}{2}$.

(2 × 5 = 10 weightage)