

THIRD SEMESTER M.Sc DEGREE EXAMINATION, NOVEMBER 2021

(Regular/Improvement/Supplementary)

STATISTICS

FMST3C12: TESTING OF STATISTICAL HYPOTHESES

Time : 3 Hours.

Maximum Weightage : 30.

Part A: All questions can be answered. Each carries two weightage.**(Ceiling 6 weightage)**

1. Distinguish between significance level and p-value.
2. Describe Type I and Type II errors associated with tests of hypotheses.
3. Explain uniformly most powerful test. Give an example.
4. Describe likelihood ratio test.
5. Describe test for goodness of fit.
6. Explain Kendall's tau.
7. Prove that SPRT terminates with probability one.

Part B: All questions can be answered. Each carries four weightage.**(Ceiling 12 weightage)**

8. A sample of size 1 is taken from $P(\lambda)$. To test $H_0 : \lambda = 1$ against $H_1 : \lambda = 2$, consider the non - randomized test

$$\varphi(x) = \begin{cases} 1 & \text{if } x > 3 \\ 0 & \text{if } x \leq 3. \end{cases}$$

Evaluate probabilities of Type I and Type II errors. Also evaluate the power of the test.

9. State and prove Neymann - Pearson fundamental lemma on tests of hypotheses.
10. Define uniformly most powerful unbiased test. Let (X_1, X_2, \dots, X_n) be a random sample from $N(\mu, \sigma^2)$ where both μ and σ^2 are unknown. For testing $H_0 : \mu = \mu_0, \sigma^2 > 0$ against $H_1 : \mu \neq \mu_0, \sigma^2 > 0$, describe whether uniformly most powerful unbiased test exists.

11. Derive the likelihood ratio test for testing $H_0 : \sigma = \sigma_0$ against $H_1 : \sigma \neq \sigma_0$ based on a random sample of size n from $N(\mu, \sigma^2)$ where both μ and σ^2 are unknown.
12. Describe clearly the Kolmogorov - Smirnov test.
13. Describe SPRT. How is it different from classical tests? Construct SPRT for testing $H_0 : \theta = \frac{1}{2}$ against $H_1 : \theta = \frac{1}{4}$ where θ is the parameter of Bernoulli distribution.
14. Derive Wald's inequality for SPRT.

**Part C: All questions can be answered. Each carries six weightage.
(Ceiling 12 weightage)**

15. a) Describe the following in the context of tests of hypotheses.
 - (i) simple and composite hypothesis
 - (ii) size of the test
 - (iii) power of the test
- b) Let (X_1, X_2, \dots, X_n) be a random sample with common pdf

$$f_{\theta}(x) = \frac{1}{\sqrt{2\theta}} e^{-\frac{|x|}{\theta}}, x \in \mathfrak{R}, \theta > 0.$$

Find a size α most powerful test for testing $H_0 : \theta = \theta_0$ against $H_1 : \theta = \theta_1$ ($\theta_1 > \theta_0$).

16. a) Let (X_1, X_2, \dots, X_n) be a random sample from $P(\lambda)$. Find a uniformly most powerful unbiased size α test for the null hypothesis $H_0 : \lambda \leq \lambda_0$ against $H_1 : \lambda > \lambda_0$.
- b) What are the properties of likelihood ratio test?
17. a) Describe Wilcoxon's signed rank test.
- b) Briefly explain the chi-square test for independence.
18. a) For an SPRT of strength (α, β) with usual notation prove that $A \leq \frac{1-\beta}{\alpha}$ and $B \geq \frac{\beta}{1-\alpha}$.
- b) Describe the following terms;
 - (i) ASN
 - (ii) OC function