D3AST2001 **(2 Pages)** **Name..............................**
Reg.No...........................

THIRD SEMESTER M.Sc. DEGREE EXAMINATION, NOVEMBER 2021
(Regular/Improvement/Supplementary)

STATISTICS
FMST3C11 - STOCHASTIC PROCESSES

Time: Three Hours **Maximum Weightage: 30**

Part A: *All* questions can be answered. Each carries *two* weightage (Ceiling 6 weightage).

1. Explain a process with stationary and independent increments.
2. Define 'period' of a state of a Markov Chain. Give an example of a MC with three states wherein all are periodic with period 2.
3. Write down the postulates of a Poisson process.
4. Describe a conditional mixed Poisson process.
5. Define renewal process and regenerative process. Give one example for each.
6. State and prove the integral equation satisfied by the renewal function.
7. Distinguish between weakly stationary and strictly stationary processes.

Part B: *All* questions can be answered. Each carries *four* weightage (Ceiling 12 weightage).

8. Define recurrent and transient states of a Markov chain. Prove that state i is recurrent if $\sum_{n=1}^{\infty} {P_{ii}}^n = \infty$ and it is transient if $\sum_{n=1}^{\infty} {P_{ii}}^n < \infty$.
9. What do you mean by stationary distribution of a Markov chain? For the three state Markov chain with transition probability matrix $\begin{bmatrix} 0 & 2/3 & 1/3 \\ 1/2 & 0 & 1/2 \\ 1/2 & 1/2 & 0 \end{bmatrix}$ find the stationary distribution.
10. Define Poisson process. If $\{N(t), t \geq 0\}$ is a Poisson process obtain the conditional distribution of N(s) given N(t)= n , s<t.
11. Obtain the mean, variance and covariance of a compound Poisson process.
12. If $\{X(t)\}$ is a birth-death process with birth rate $\lambda_n = \lambda$ and $\mu_n = \mu$, then calculate $E\{X(t)/X(0) = t\}$.
13. Define birth and death process. Obtain the forward Kolmogorov differential equations.
14. Describe the basic characteristics of a queuing model.

(P.T.O.)

Part C: *All* questions can be answered. Each carries *six* weightage (Ceiling 12 weightage).

15. Define a Galton Watson branching process. Obtain the mean and variance functions of such a process.
16. (a) Show that the sum of two independent Poisson processes is again a Poisson process.

    (b) If $\{N(t), t \geq 0\}$ is a Poisson process, obtain the joint distribution of the arrival times given N(t)= n.
17. State and prove elementary renewal theorem.
18. Develop the embedded Markov chain technique for the M/G/1 queueing system and obtain its steady state solution.