D3AMT2004 **(2 Pages)** **R**eg. No: ·················
Name: ·················

THIRD SEMESTER M.Sc DEGREE EXAMINATION, NOVEMBER 2021

(Regular/Improvement/Supplementary)

MATHEMATICS

FMTH3C14: PDE AND INTEGRAL EQUATIONS

Time : 3 Hours. **Maximum Weghtage : 30.**

Part A: All questions can be answered. Each carries one weightage. (Ceiling 6 weightage)

1. Give example of a quasi linear first order partial differential equation that is not a linear equation. Give proper reason.
2. Define the term "Characteristic Curve" of a first order quasilinear partial differential equation with one example.
3. Explain the term "Monge Cone" associated with a general first order partial differential equation.
4. Explain the classification of second order linear partial differential equations. Give example of an elliptic type equation.
5. Write the one dimensional wave equation and explain the boundary/initial conditions.
6. State the maximum principle for heat equation.
7. Write the different types of integral equations and give one example for each.
8. What is the meaning of a Green's function for an integral equation ?

Part B: All questions can be answered. Each carries two weightage. (Ceiling of 12 weightage)

9. Solve the equation $u_x + u_y = 2$ subject to the initial condition $u(x,0) = x^2$ by the method of characteristics.
10. Show that the Cauchy Problem $u_x + u_y = 1,\ u(x,x) = x$ has infinitely many solutions.
11. Prove that the equation $x^2u_{xx} - 2xyu_{xy} + y^2uyy + xu_x + yu_y = 0$ is parabolic and find its canonical form. Also find the general solution on the half-plane $x > 0$.

12. State and prove a necessary condition for the existence of a solution to the Neumann problem.

13. Define a harmonic function in a domain.
If u is harmonic in a domain D, prove that $-u$ also is harmonic in D.

14. State and prove the strong maximum principle.

15. Prove that $\int_a^x \int_a^{x_n} \cdots \int_a^{x_3} \int_a^{x_2} f(x_1) dx_1 dx_2 \cdots dx_n = \frac{1}{(n-1)!} \int_a^x (x-\xi))^{n-1} f(\xi) d\xi.$

16. Transform the boundary value problem $\frac{d^2y}{dx^2} + \lambda y = 0, \quad y(0) = 1, \quad y(l) = 0$ into an integral equation.

17. Prove that the characteristic functions corresponding to two different characteristic numbers λ_m and λ_n of a Homogeneous Fredholm integral equation $y(x) = \lambda \int_a^b K(x,\xi) y(\xi) d\xi$ with a symmetric kernel $K(x,\xi)$ are orthogonal.

Part C: All questions can be answered. Each carries six weightage. (Ceiling 12 weightage)

18. Solve the equation $u_x + u_y + u = 1$,
subject to the initial condition $u = \sin x$, on $y = x + x^2$, $x > 0$.

19. a) Explain the d'Alembert's method of solution of a Cauchy's problem for the one-dimensional homogeneous wave equation.
    b) Solve the Cauchy problem $u_{tt} - 9u_{xx} = 0, \quad -\infty < x < \infty, \quad t > 0$

$$u(x,0) = f(x) \begin{cases} 1 & \text{if } |x| \le 2 \\ 0 & \text{if } |x| > 2 \end{cases} \qquad u_t(x,0) = g(x) \begin{cases} 1 & \text{if } |x| \le 2 \\ 0 & \text{if } |x| > 2 \end{cases}$$

20. Describe the heat conduction problem in a finite interval. Also describe the method of separation of variables to find the solution of the heat equation.

21. Solve the integral equation $y(x) = \lambda \int_0^1 (1 - 3x\xi) y(\xi) d\xi + F(x)$ by the method of separable kernel.