D3AMT2002 **(2 Pages)** **Name..............................**
Reg. No..........................

THIRD SEMESTER M. Sc. DEGREE EXAMINATION, NOVEMBER 2021
(Regular/Improvement/Supplementary)

MATHEMATICS
FMTH3C12- COMPLEX ANALYSIS

Time: Three Hours **Maximum Weightage: 30**

Part A: All questions can be answered. Each carries one weightage (Ceiling 6 weightage).

1. What is Spherical representation of complex numbers?
2. Define a path, smooth path and piecewise smooth path. Give examples as diagrams.
3. Prove that the reflection $z \to \overline{z}$ is not a linear transformation.
4. Define the line integral $\oint_g f(z)dz$ of a function $f(z)$ along a rectifiable path g. Further show that $\oint_{-g} f(z)dz = -\oint_g f(z)dz.$
5. Define zero of an analytic function. Find all zeroes of $\sin z$.
6. State the first version and homotopic versionof Cauchy's theorem
7. State whether true or false the statement "The singularity of $\frac{\sin(z-1)}{z-1}$ at $z=1$ is a pole". Justify your claim.
8. Using Cauchy's residue theorem evaluate $\oint_g \frac{5z}{(z-1)}dz$ where g is the positively oriented circle $|z|=2$.

Part B: All questions can be answered. Each carries two weightage (Ceiling 12 weightage).

9. Find the subset of S which correspond to the real and imaginary axes of $\mathbf{C}$.
10. If $\sum a_n$ converges absolutely, then show that $\sum a_n$ converges.
11. Show that the cross-ratio of four points is real if and only if the points lie on circle or a straight line.
12. Derive the Cauchy's estimate.
13. State and prove Liouville's therem.
14. State and prove the first version of Cauchy's integral formula.

(P.T.O.)

15. Define singular point of a function $f(z)$, and explain various types of singularities with one example each.

16. State and prove Cauchy's Residue Theorem.

17. State and prove Rouche's theorem.

Part C: All questions can be answered. Each carries six weightage (Ceiling 12 weightage).

18. a) Derive a set of necessary and sufficient conditions for $f(z) = u + iv$ to be analytic at a point.

    b) Define the cross-ratio of four points and show that it is invariant under a Mobius transformation.

19. a) State and prove maximum modulus principle.

    b) State and prove open mapping theorem.

20. a) If f has an isolated singularity at a, prove that $z = a$ is a removable singularity if and only if $\lim_{z \to a} (z - a) f(z) = 0$.

    b) State and prove Casorati-Weierstrass theorem on essential singularities.

21. a) If $f(z)$ is meromorphic in a region G with the zeroes z_j and the poles p_k counted according to multiplicity, show that $\frac{1}{2\pi i}\int_\gamma \frac{f'(z)}{f(z)}dz = \sum_j n(\gamma, z_j) - \sum_k n(\gamma, p_k)$ for every closed rectifiable curve $\gamma \approx 0$ in G, which does not pass through any of the zeroes or poles.

    b) Using Residue Theorem, choosing a suitable contour, evaluate $\oint_{-\infty}^{+\infty} \frac{dx}{(x^2+1)}$.