

## THIRD SEMESTER M.Sc. DEGREE EXAMINATION, NOVEMBER 2020

## STATISTICS

## FMST3C12: TESTING OF STATISTICAL HYPOTHESES

Time: 3 Hours

Maximum Weightage: 30

**Part A: Answer any *four* questions. Each carries 2 weightage.**

1. Distinguish between parametric hypothesis and non-parametric hypothesis. Illustrate through examples.
2. What is p-value? How is it related to significance level?
3. Describe uniformly most powerful unbiased test. Give an example.
4. Bringout the connection between likelihood ratio test and sufficient statistic.
5. Distinguish between parametric and non-parametric tests.
6. Describe the chi-square test for homogeneity.
7. Define Operating Characteristic (OC) function. What is it used for?

**(4 × 2 = 8 Weightage)****Part B: Answer any *four* questions. Each carries 3 weightage.**

8. Describe Type I and Type II errors in tests of hypotheses. A sample of size 1 is taken from exponential distribution with parameter  $\theta$ . To test  $H_0 : \theta = 1$  against  $H_1 : \theta > 1$ , the test to be used is the non-randomized test

$$\varphi(x) = \begin{cases} 1 & \text{if } x > 2 \\ 0 & \text{if } x \leq 2. \end{cases}$$

Derive the power function? Find the size of the test.

9. Find the Neymann - Pearson size  $\alpha$  test of  $H_0 : \beta = 1$  against  $H_1 : \beta = \beta_1 (> 1)$  based on a sample of size  $n$  from

$$f(x, \beta) = \begin{cases} \beta x^{\beta-1} & 0 < x < 1 \\ 0 & \text{otherwise.} \end{cases}$$

10. When do you say that a test has MLR? Does the Laplace family of pdf

$$f(x, \theta) = \frac{1}{2}e^{-|x-\theta|}, \quad -\infty < x < \infty, \theta \in \mathfrak{R}.$$

possess an MLR?

11. Define the following terms;
- (i) unbiased test (ii) MP test
  - (iii)  $\alpha$  - similar test and (iv) invariant test
12. Describe the following nonparametric tests;
- (i) Wilcoxon signed rank test
  - (ii) Kolmogorov - Smirnov test
  - (iii) Median test
13. For an SPRT with stopping bounds  $A$  and  $B$  ( $A > B$ ) and strength  $(\alpha, \beta)$ , prove that  $A \leq \frac{1-\beta}{\alpha}$  and  $B \geq \frac{\beta}{1-\alpha}$ .
14. State and prove Wald's inequality.

(4 × 3 = 12 Weightage)

**Part C: Answer any two questions. Each carries 5 weightage.**

15. a) Describe the following terms;
- (i) critical region (ii) power function (iii) power of a test
- b) m State and prove Neymann - Pearson fundamental lemma on tests of hypotheses.
16. a) Find a uniformly most powerful size  $\alpha$  test of  $H_0 : \theta \leq \theta_0$  against  $H_1 : \theta > \theta_0$  based on a sample of  $n$  observations from a population with pdf

$$f_{\theta}(x) = \frac{1}{\sqrt{2\pi}}e^{-\frac{(x-\theta)^2}{2}}, \quad -\infty < x < \infty, -\infty < \theta < \infty.$$

- b) Let the power function of a test  $\varphi$  of  $H_0 : \theta \in \Theta_0$  against  $H_1 : \theta \in \Theta_1$  be continuous in  $\theta$ . Prove that a UMP  $\alpha$  - similar test is UMP unbiased, provided its size is  $\alpha$  for testing  $H_0$  against  $H_1$ .
17. a) Describe two sample Kolmogorov - Smirnov test. Is it distribution free? Justify your answer.
- b) Describe chi-square test for goodness of fit.
18. a) Describe how do you test a simple null hypothesis  $H_0$  against simple alternative hypothesis  $H_1$  using SPRT.
- b) Prove that SPRT terminates with probability one.

(2 × 5 = 10 Weightage)