D3AST1901 (2 Pages) **Name............................**
Reg. No..........................

THIRD SEMESTER M. Sc. DEGREE EXAMINATION, NOVEMBER 2020
STATISTICS
FMST3C11- STOCHASTIC PROCESSES

Time: Three Hours **Maximum Weightage: 30**

Part A: Answer any *four* questions. Each carries *two* weightage.

1. Define a stochastic process, its state space and parameter space. Illustrate them with a suitable example.
2. Define periodicity of a state of a Markov chain and show that it is a class property.
3. Write down the postulates of Poisson process.
4. Describe compound Poisson process.
5. Define renewal process and give an example.
6. Describe renewal reward process.
7. What is Little's formula in queueing theory? State its relevance.

(4 × 2 = 8 weightage)

Part B: Answer any *four* questions. Each carries *three* weightage.

8. "In a finite irreducible Markov chain all states are recurrent". Prove or disprove this statement.
9. Obtain the Chapman-Kolmogorov equation satisfied by the transition probabilities of a discrete parameter Markov chain. What is its use?
10. Derive the expression for $P(X(t) = n)$ for the Poisson process $\{ X(t), t \geq 0\}$.
11. For a homogeneous Poisson process X(t) with rate λ , derive the correlation coefficient between X(t) and X(t+s), t,s >0.
12. Prove that the renewal function satisfies the equation, $M(t) = F(t) + \int_0^t M(t-x)dF(x)$.
13. Show that linearity of the renewal function characterizes a Poisson process.
14. Describe M/M/1 queueing model and obtain its steady state solution.

(4 × 3 = 12 weightage)

(P.T.O.)

Part C: Answer any *two* questions. Each carries *five* weightage.

15. Define Galton Watson branching process. In the usual notations, show that for a branching process $\{X_n; n \geq 0\}$ with $X_0 = 1$, $P_n(s) = P_{n-1}(P(s))$ and $P_n(s) = P(P_{n-1}(s))$.

16. Prove that $\{N(t), t \geq 0\}$ is a Poisson process if, and only if, the successive interarrival times are i.i.d exponentially distributed.

17. State and prove elementary renewal theorem.

18. Describe a M/G/1 queueing model and verify whether the process X(t) that represents the number of customers in the system at time t is a Markov process or not. Obtain the Pollaczek- Khintchine formula for this model.

(2 × 5 = 10 weightage)