D3AMT1904 **(2 Pages)** **Name.............................**
Reg.No..........................

THIRD SEMESTER M. Sc. DEGREE EXAMINATION, NOVEMBER 2020
MATHEMATICS
FMTH3C14- PDE AND INTEGRAL EQUATIONS

Time: Three Hours **Maximum Weightage: 30**

Part A: Answer *all* questions. Each carries *one* weightage.

1. Define quasilinear partial differential equation. Give an example.
2. Show that there are infinitely many solutions for the Cauchy problem

   $u_x + u_y = 1,\ u(x,x) = x.$
3. Show that the following equation is hyperbolic:

   $u_{xx} + 6u_{xy} - 16u_{yy} = 0.$
4. Write d'Alembert's formula for the one dimensional homogenous wave equation.
5. Let u(x, t) be a solution of the wave equation $u_{tt} - c^2u_{xx} = 0$, which is defined in the whole plane. Assume that *u* is constant on the line $x = 2 + ct$. Prove that $u_t + cu_x = 0$.
6. Define Dirichlet problem and Neumann Problem.
7. Write Fredholm equation.
8. Define Kernel of an integral equation.

(8 × 1 = 8 weightage)

Part B: Answer any *two* questions from each unit. Each carries *two* weightage.

Unit 1

9. Using method of characteristic solve $u_x + u_y = 2$ subject to the initial condition $u(x,0) = x^2$.
10. Solve the eikonal equation $u_x^2 + u_y^2 = n^2$, where the surface $u = c$, are the wavefronts, and n is the refraction index of the medium.
11. Prove that the type of a linear second-order PDE in two variables is invariant under a change of coordinates.

Unit 2

12. Solve $u_{tt} - 4u_{xx} = 0 \quad ; \quad 0 < x < 1 ; t > 0$

$$u_x(0,t) = u_x(1,t) = 0 ; \quad t \geq 0,$$
$$u(x,0) = f(x) = \cos^2 \pi x \quad ; \quad 0 \leq x \leq 1,$$
$$u_t(x,0) = g(x) = \sin^2 \pi x \cos \pi x ; \ 0 \leq x \leq 1.$$

(P.T.O.)

13. Solve the equation $u_t = 17u_{xx}$; $0 < x < \pi$; $t > 0$; with the boundary conditions $u(0,t) = u(\pi,t) = 0$; $t \geq 0$, and the initial conditions $u(x,0) = \begin{cases} 0 & 0 \leq x \leq \pi/2 \\ 2 & \pi/2 < x < \pi \end{cases}$

14. Let D be a bounded domain, and let $u(x,y) \in C^2(D) \cap C(\overline{D})$ be a harmonic function in D. Then prove that the maximum of u in $\overline{D}$ is achieved on the boundary ∂D.

Unit 3

15. Prove that $\int_a^x \int_a^{x_n} \ldots \int_a^{x_3} \int_a^{x_2} f(x_1)\, dx_1 dx_2 \ldots dx_{n-1} dx_n = \frac{1}{(n-1)!} \int_a^x (x-\xi)^{n-1} f(\xi) d\xi$

16. Solve $\left.\begin{aligned} \frac{d^2y}{dx^2} + \lambda y = f(x), \\ y(0) = 1, \quad y'(0) = 0 \end{aligned}\right\}$

17. Solve $\mathcal{L}\, y = y''$, $y(0) = y(l) = 0$.

(6 × 2 = 12 weightage)

Part C: Answer any *two* questions. Each carries *five* weightage.

18. (a) Solve the equation $u_x + 3y^{2/3}u_y = 2$ subject to the initial condition

$u(x,1) = 1 + x$.

(b) Solve the equation $(y+u)u_x + yu_y = x - y$ subject to the initial conditions

$u(x,1) = 1 + x$.

19. Suppose that $L[u] = au_{xx} + 2bu_{xy} + cu_{yy} + du_x + eu_y + f_u = g$ is hyperbolic in a domain D. There exists a coordinatesystem (ξ, η) in which the equation has the canonical form $w_{\xi\eta} + l[w] = G(\xi,\eta)$, where $w(\xi,\eta) = u(x(\xi,\eta), y(\xi,\eta))$, l_1 is a first-order linear differential operator and G is a function which depends on $L[u] = au_{xx} + 2bu_{xy} + cu_{yy} + du_x + eu_y + f_u = g$.

20. Solve the heat conduction problem

(a) $u_t - ku_{xx} = 0$; $0 < x < L, t > 0$,

(b) $u(0,t) = u(L,t) = 0$; $t \geq 0$,

(c) $u(x,0) = f(x)$; $0 \leq x \leq L$

21. Solve the Laplace equation in the rectangle $0 < x < b$, $0 < y < d$, subject to the Dirichlet boundary conditions $u(0,y) = f(y)$; $u(b,y) = g(y)$; $u(x,0) = 0$; $u(x,d) = 0$.

(2 × 5 = 10 weightage)