D3AMT1903 (2 Pages) **Name.............................**
Reg.No..........................

THIRD SEMESTER M. Sc. DEGREE EXAMINATION, NOVEMBER 2020
MATHEMATICS
FMTH3C13- FUNCTIONAL ANALYSIS

Time: Three Hours **Maximum Weightage: 30**

Part A: Answer *all* questions. Each carries *one* weightage.

1. Give an example of an infinite dimensional linear space. Justify it.
2. Define a norm on the space C[0,1], and verify that it is indeed a norm.
3. Show that the sequence space λ_1 is proper subspace of λ_2.
4. Show that the inner product is a continuous function with respect to both variables.
5. If x is orthogonal to every member of a complete system in a Hilbert space, then prove that $x = 0$.
6. If E is a closed subspace of a Hilbert space with $co\dim E = 1$ then prove that the subspace $E^{\perp}$ is one dimensional.
7. For any x_1, x_2 in a normed space with $x_1 \neq x_2$, prove that there exists a bounded linear functional f such that $f(x_1) \neq f(x_2)$.
8. Show that every bounded operator of finite rank is a compact operator.

(8 × 1 = 8 weightage)

Part B: Answer any *two* questions from each unit. Each carries *two* weightage.

Unit 1

9. Show that the linear space of all sequences with finite support (i.e., the sequences with all but finite zero elements) is isomorphic to the space of all polynomials.
10. If X is a Banach space and if E is a closed subspace of X, show that the quotient space X / E is a Banach space.
11. Define the completion of a normed space and show that it is unique up to isometry.

(P.T.O.)

Unit 2

12. State and prove Bessel's inequality.

13. If f is a linear functional on a normed space and if $\ker f$ is closed, show that f is bounded.

14. If L is a closed subspace of a Hilbert space H, then prove that $H = L \oplus L^{\perp}$.

Unit 3

15. Show that the dual space of c_0 is λ_1.

16. If X is a normed space and if Y is a complete normed space, prove that the space $L(X \alpha \; Y)$ is a Banach space.

17. Define the dual operator $A*$ of $A : X \to Y$. If A is compact, show that $A*$ is also compact.

(6 × 2 = 12 weightage)

Part C: Answer any *two* questions. Each carries *five* weightage.

18. State and prove Holders inequality for scalar sequences. Use it to deduce the Minkowski's inequality.

19. (a) Show that any two separable infinite dimensional Hilbert spaces are isometrically equivalent.

    (b) State and prove Riesz representation theorem.

20. (a) Prove that $\lambda_1^* = \lambda_\infty$.

    (b) Define compact operator. Show that the space $K(X \alpha \; Y)$ is a closed subspace of $L(X \alpha \; Y)$.

21. (a) Show that every finite dimensional normed space is a reflexive space.

    (b) Define the three different notions of convergence in the space of bounded operators. Distinguish these using examples.

(2 × 5 = 10 weightage)