D3AMT1902 **(2 Pages)** **Name..............................**
Reg.No...........................

THIRD SEMESTER M. Sc. DEGREE EXAMINATION, NOVEMBER 2020
MATHEMATICS
FMTH3C12- COMPLEX ANALYSIS

Time: Three Hours **Maximum Weightage: 30**

Part A: Answer *all* questions. Each carries *one* weightage.

1. Define an analytic function. Show that the function $f(z)=\overline{z}$ is nowhere analytic.
2. Define Riemann Stieltjes integral.
3. Define a Mobius transformation. Find the fixed points of the linear transformations $w=\frac{z}{2z-1}$
4. If $T_1z=\frac{z+2}{z+3}$ and $T_2z=\frac{z}{z+1}$, find T_1T_2z, T_2T_1z and $T_1^{-1}T_2z$
5. Find the winding number $n(g,3)$ where g is a positively oriented circle $|z-1|=2$. with a valid explanation.
6. State whether true or false the statement "The singularity of $\frac{\sin(z-1)}{z-1}$ at $z=1$ is a pole". Justify your claim.
7. Define a pole of a function and its order. Give an example specifying the pole and its order.
8. Give one example each for a
   a) function with a removable singularity at $1+i$
   b) function with an essential singularity at $1+i$

(8 × 1 = 8 weightage)

Part B: Answer any *two* questions from each unit. Each carries *two* weightage.

Unit 1

9. Define the cross-ratio of four points and show that it is invariant under a Mobius transformation.
10. State and prove the symmetry principle.
11. If $g:[a,b]\to c$ is piecewise smooth then show that it is of bounded variation and $V(g)=\oint_a^b|g'(t)|dt$.

(P.T.O.)

Unit 2

12. Let f be analytic in the disk $B(a,R)$ and suppose g is a closed rectifiable curve in $B(a,R)$. Then show that $\oint_g f = 0$.

13. Show that the zeroes of an analytic function are isolated.

14. Define the winding number $n(g,a)$ and show that $n(g,a)$ is constant in each of the regions determined by g and zero in the unbounded region.

Unit 3

15. State and prove Casorati-Weierstrass theorem on essential singularities.

16. State and prove Rouche's theorem.

17. State and prove Schwarz's lemma.

(6 × 2 = 12 weightage)

Part C: Answer any *two* questions. Each carries *five* weightage.

18. a) Show that a non-constant analytic function maps open sets into open sets

    b) What is a conformal mapping? Show that A mapping by an analytic function is conformal at all points where it has a non-vanishing derivative

19. a) Show that an analytic function is infinitely differentiable.

    b) Derive the formula for the nth derivative of an analytic function.

20. a) If f is analytic and non-constant in G, $a \in G$ is such that $f(a)=0$, show that there exists an $R>0$ such that $B(a,R) \subseteq G$ and $f \neq 0$ for $0<|z-a|<R$.

    b) If $\gamma:[0,1] \to \mathbf{C}$ is a closed rectifiable curve which does not pass through the point a then show that $\int_{\gamma} \frac{dz}{z-a}$ is a multiple of $2\pi i$.

21. a) Define a convex function. Derive a necessary and sufficient condition for a function $f:[a,b] \to \mathbf{R}$ to be convex.

    b) State and prove Hadamard's three circle theorem.

(2 × 5 = 10 weightage)