

D2AST2402

Name.....

Reg. No.

SECOND SEMESTER M.Sc. DEGREE EXAMINATION, APRIL 2025
(Regular/Improvement/Supplementary)
STATISTICS
FMST2C07: REGRESSION ANALYSIS

Time : 3 Hours

Maximum Weightage: 30

Part A: Answer *any four* questions. Each carries 2 weightage.

1. Define coefficient of determination.
2. Consider the regression model $E[Y_i] = \beta_0 + \beta_1 x_i + \beta_2(3x_i^2 - 2)$ ($i = 1, 2, 3$), where $x_1 = -1$, $x_2 = 0$, and $x_3 = +1$. Find the least squares estimates of β_0 , β_1 , and β_2 . Show that the least squares estimates of β_0 and β_1 , are unchanged if $\beta_2 = 0$.
3. Define hat matrix and state its properties.
4. Discuss non-parametric regression models.
5. Distinguish between link functions and linear predictors.
6. Discuss the assumptions of the Poisson regression model.
7. Why is unequal variance a problem? Explain about different variance stabilizing transformations.

(4 x 2 = 8 weightage)

Part B: Answer *any four* questions. Each carries 3 weightage.

8. Suppose that $Y \sim N_n(X\beta, \sigma^2 I_n)$, where X is $n \times p$ of rank p .
 - (i) Find $Var(S^2)$.
 - (ii) Evaluate $E(Y'A_1Y - \sigma^2)^2$ for $A_1 = \frac{1}{n-p+2}[I_n - X(X'X)^{-1}X']$.
 - (iii) Prove that $Y'A_1Y$ is an estimate of a σ^2 with a smaller mean-squared error than S^2 .
9. Explain the situations in which ordinary least square estimates are not efficient. State and prove Gauss-Markov theorem in the case of generalized least square method.
10. Explain the concept of autocorrelation and list the commonly used test procedures for detecting the presence of autocorrelation.
11. What are the different methods used in the diagnostics of the leverage and influential observations?

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12. What is the need for piecewise polynomial fitting? Discuss the method of splines in this context.
13. Differentiate between parametric and non-parametric regression procedures. What are the usual estimation procedures used in non-parametric regression?
14. Explain the parametric estimation and inferential problems on GLM.
(4 x 3 = 12 weightage)

Part C: Answer *any two* questions. Each carries 5 weightage.

15. Discuss various scaled residuals. Explain the model adequacy checking using residuals.
16. If $Y \sim N_n(X\beta, \sigma^2 I_n)$, where X is $n \times p$ of rank p , then show that:
 - (i) $\hat{\beta} \sim N_p(\beta, \sigma^2(X'X)^{-1})$
 - (ii) $\frac{(\hat{\beta}-\beta)'(X'X)(\hat{\beta}-\beta)}{\sigma^2} \sim \chi_{(p)}^2$
 - (iii) $\hat{\beta}$ is independent of $S^2 = \frac{RSS}{n-p}$
 - (iv) $\frac{RSS}{\sigma^2} = \frac{(n-p)S^2}{\sigma^2} \sim \chi_{(n-p)}^2$.
17. Explain logistic regression models. Estimate the parameters of the model.
18. Explain the problem of ill-conditioning in polynomial regression. Describe how orthogonal polynomials can be used to overcome the ill-conditioning. Explain the fitting of orthogonal polynomials.

(2 x 5 = 10 weightage)