D2AST2401 (2 Pages) Name................................
Reg.No............................

SECOND SEMESTER M.Sc. DEGREE EXAMINATION, APRIL 2025
(Regular/Improvement/Supplementary)

STATISTICS
FMST2C06- ESTIMATION THEORY

Time: 3 Hours **Maximum Weightage: 30**

Part A: Answer any *four* questions. Each carries 2 weightage.

1. Define one parameter exponential family. Show that beta distribution of first kind with parameters p and q belongs to the two parameter exponential family. Also find joint sufficient statistic for p and q.
2. State and prove factorization theorem.
3. State the invariance property of consistent estimators. Let $X_1, X_2, \dots, X_n$ be a random sample from the Poisson distribution with mean λ , obtain a consistent estimator for $e^{-\lambda}$.
4. Describe the method of percentiles for finding consistent estimator.
5. Let $X_1, X_2, \dots \dots, X_n$ be a random sample from an exponential distribution with location parameter μ and scale parameter σ. Determine moment estimators for μ and σ.
6. Define one parameter Cramer family. Give an example of a family which is not an exponential family but is a member of Cramer family of distributions.
7. Explain the construction of shortest length confidence interval.

(4 × 2 = 8 weightage)

Part B: Answer any *four* questions. Each carries 3 weightage.

8. Let $X_1, X_2, \dots, X_n$ is a random sample from $U(\theta - \frac{1}{2}, \theta + \frac{1}{2})$ population. Find a sufficient statistic for θ. Is it complete?

9. a) State and prove Basu's theorem.
   b) X_i's are i.i.d. N (μ, σ^2) where σ^2 is known. Show that $(\bar{X}, S^2)$ is independent.

10. a) State and prove Cramer Rao inequality.
    b) Let $X_1, X_2, \dots, X_n$ is a random sample from a population with p.d.f
    $$f(x, \theta) = \frac{1}{\pi} \frac{1}{\{1 + (x - \theta)^2\}}, -\infty < x < \infty.$$
    Find CRLB for the variance of the unbiased estimate of θ.

11. Define Consistent Asymptotically Normal (CAN) Estimators. Let $X_1, X_2, \dots, X_n$ be a random sample from $U(0, \theta)$, $\theta > 0$. Show that: (i) $X_{(n)}$ is the MLE of θ (ii) $X_{(n)}$ is not CAN (iii) $X_{(n)}$ is consistent.

12. Show that under certain regularity conditions, MLE is consistent.

(P.T.O.)

13. a) State and prove invariant property of MLE.

b) Give an example to prove that MLE's are not always unbiased.

14. A random sample of size n is taken from exponential distribution with mean λ . Find a two sided confidence interval for λ based on the pivot $2\sum_{i=1}^{n} X_i / \theta$.

(4 × 3 = 12 weightage)

Part C: Answer any *two* questions. Each carries 5 weightage.

15. a) Show that uniformly minimum variance unbiased estimator (UMVUE) if it exists, is unique.

b) Let $X_1, X_2, \dots, X_n$ be a random sample from Poisson (λ).
Find the UMVUE of: (i) λ (ii) $e^{-\lambda}$ (iii) λ^2 (iv) $\lambda^2 + 2\lambda$ (v) $\lambda e^{-\lambda}$.

16. a) Find a minimal sufficient statistic for (α, β) with p. d. f.

$$f(x; \alpha, \beta) = \frac{\alpha^{\beta}}{\Gamma\alpha} e^{-\alpha x}\, x^{\beta-1}, x, \alpha, \beta > 0$$

b) Let $X_1, X_2, \dots, X_n$ are i.i.d. U (0, θ). θ > 1.

Define Y_i = 1, if X_i > 1
0, elsewhere.

Find the estimate of θ by the method of moments.

17. a) Show that the Bayes estimator of θ under the squared error loss function is the mean of the posterior distribution.

b) Let $X_1, X_2, \dots, X_n$ be a random sample from an exponential distribution with parameter θ. For estimating θ using the squared error loss function, a prior density of θ is given,

$$\pi(\theta) = e^{-\theta}, \theta > 0$$

18. a) Explain Bayesian and Fiducial interval.

b) Let $X_1, X_2, \dots, X_n$ be a random sample of size drawn from a Poisson distribution with parameter λ. Assuming that the prior distribution of λ is Gamma (α, β). Find 100 (1 − α) % Bayesian confidence interval for λ. Compare it with classical shortest length confidence interval.

(2 × 5 = 10 weightage)