D2AMT2403 (2 Pages) Name ....

Reg.No..........................

SECOND SEMESTER M.Sc. DEGREE EXAMINATION, APRIL 2025
(Regular/Improvement/Supplementary)

MATHEMATICS
FMTH2C08: TOPOLOGY

Time: 3 Hours **Maximum Weightage: 30**

Part A: Answer *all* questions. Each carries 1 weightage.

1. Explain Sierpinski space. Prove that the Sierpinski space is not obtainable from a pseudometric.
2. Let (X, T) be a topological space and let A⊂X. Then prove that $\bar{\bar{A}} = \bar{A}$.
3. Define co-finite topology and prove that every cofinite space is compact.
4. When do we say that a topological property is divisible? Prove that being a finite space is divisible.
5. In a Hausdorff space, prove that limits of sequences are unique.
6. Prove that compactness is preserved under continuous functions.
7. Write an example of a Hausdorff topology on the set X= {1, 2, 3}.
8. Prove that every compact Hausdorff space is a T_3 space.

(8 × 1 = 8 weightage)

Part B: Answer any *two* questions from each unit. Each carries 2 weightage.

Unit 1

9. Prove that the real line with the semi-open interval topology is not second countable.
10. Prove that intersection of two open sets in a metric space is open.
11. Prove that int(A) , A⊂X is the union of all open sets contained in A.

Unit 2

12. Prove that the co-countable topology on a set makes it into a Lindelof space.
13. Prove that every closed, surjective map is a quotient map.
14. Prove that the product topology is the weak topology determined by the projection functions.

(P.T.O.)

Unit 3

15. Prove that a space X is T_1 space iff every finite subset of X is closed.

16. Prove that a compact subset of a Hausdorff space is closed.

17. Prove that there can be no continuous one-to-one map from the unit circle S^1 into the real line.

(6 × 2 = 12 weightage)

Part C: Answer any *two* questions. Each carries 5 weightage.

18. a) Prove that the usual topology on the Euclidean plane R^2 is strictly weaker than the topology induced by lexicographic ordering.

    b) Determine the topology induced by a discrete metric on a set.

19. a) Prove that if the space (X,T) has a base B of cardinality α, then the cardinality of T cannot exceed 2^{α}.

    b) Prove that a subset A of a space X is dense in X if and only if for every non-empty open subset B of X, $A \cap B \neq \emptyset$.

20. a) Let (X,d) be a compact metric space and U be an open cover of X. Then prove that there exists a positive real number *r* such that for any $x \in X$, there exists $V \in U$ such that $B(x, r) \subset V$.

    b) Prove that every second countable space is first countable.

21. State and prove the Tietze extension theorem.

(2 × 5 = 10 weightage)