D2AMT2402 **Reg.No........................**

Name: ....

SECOND SEMESTER M.Sc. DEGREE EXAMINATION, APRIL 2025
(Regular/Improvement/Supplementary)
MATHEMATICS
FMTH2C07 - REAL ANALYSIS II

Time: 3 Hours **Maximum Weightage: 30**

Part A: Answer *all* questions. Each carries 1 weightage.

1. State and prove monotonicity property of outer measure.
2. Prove that if $m^*(A) = 0$, then $m^*(A \cup B) = m^*(B)$.
3. Show that every monotone function that is defined on an interval is measurable.
4. Suppose f and g are real valued functions defined on real numbers, and suppose f is measurable and g is continuous. Is the composition $f \circ g$ necessarily measurable? Give reason.
5. Is every bounded measurable function defined on measurable set is integrable ? Justify your answer.
6. Show that $L^\infty(X, \mu)$ is a normed linear space over the real numbers.
7. Define total variation. Also show that the linear combination of two functions of bounded variation is also of bounded variation.
8. State and prove Young's Inequality.

(8 x 1= 8 weightage)

Part B:Answer any *two* questions from each unit. Each carries 2 weightage.

Unit I

9. Show that union of finite collection of measurable sets is measurable.
10. Show that a set E is measurable if and only if for each $\epsilon > 0$ there is a closed set F and an open set O for which $F \subseteq E \subseteq O$ and $m^*(O \sim F) < \epsilon$.
11. Show that there exists a non measurable set.

(P.T.O.)

Unit II

12. State and prove Chebychev's Inequality.

13. Assume E has finite measure. Let the sequence of functions f_n be uniformly integrable over E. Show that if $f_n \to f$ pointwise a.e. on E, then f is integrable over E. Is the condition $m(E) < \infty$ necessary? Give reason.

14. State and prove Monotone convergence theorem. Also show that the Monotone convergence theorem may not hold for decreasing sequence functions.

Unit III

15. Let C be a countable subset of the open interval (a, b). Then show that there exists an increasing function on (a, b) that is continuous only at points in $(a, b) \sim C$.

16. Show that a function f is of bounded variation on the closed bounded interval $[a, b]$ if and only if it is the difference of two increasing functions on $[a, b]$.

17. Show that a function f on a closed bounded interval $[a, b]$ is absolutely continuous on $[a, b]$ if and only if it is an indefinite integral over $[a, b]$.

(6 x 2= 12 weightage)

Part C: Answer any two questions. Each carries 5 weightage

18. a) Show that class of lebesgue measurable sets form a sigma algebra. b) State and prove Lusin's Theorem.

19. a) State and prove Lebesgue dominated convergence theorem. b) State and prove Fatous lemma. Show by an example that the inequality in Fatous lemma may be strict.

20. a) Let f and g be integrable over E. Then for any α and β show that
$\int_E(\alpha f + \beta g) = \alpha \int_E f + \beta \int_E g$.
b) State and prove Lebesgue' s Theorem.

21. a) State and prove Holders inequality. When does its equality occur? b) Let E be a measurable set and $1 \leq p \leq \infty$. Then show that $L^P(E)$ is a Banach Space.

(2 x 5= 10 weightage)