D2AST2303 (1 Page) Name.............................
Reg.No..........................

SECOND SEMESTER M.Sc. DEGREE EXAMINATION, APRIL 2024
(Regular/Improvement/Supplementary)

STATISTICS
FMST2C08- PROBABILITY THEORY

Time: 3 Hours **Maximum Weightage: 30**

Part A: Answer any *four* questions. Each carries 2 weightage.

1. State Borel 0-1 law.
2. What do you mean by induced probability space?
3. Define convergence in probability.
4. Show that binomial random variable converges in law to Poisson random variable as $n \to \infty$ and $np \to \lambda$.
5. Show that characteristic function of Laplace pdf is a constant multiple of Cauchy pdf.
6. What do you mean by Doob,s decomposition of submartingale?
7. Discuss super and sub martingale.

(4 × 2 = 8 weightage)

Part B: Answer any *four* questions. Each carries 3 weightage.

8. Define and discuss conditional probability measure.
9. State and prove basic inequality.
10. State Dominated Convergence theorem.
11. If X_n converges to X as and Y_n converges to Y as then show that X_n+Y_n converges as to X+Y.
12. If P[(X,Y)=(1,1)]=1/3= P[(X,Y)=(1,1)] and P[(X,Y)=(-1,1)]=1/6= P[(X,Y)=(-1,-1)], find the characteristic function of X+Y.
13. Discuss the invariance principle of characteristic function.
14. State Radon-Nikodym theorem.

(4 × 3 = 12 weightage)

Part C: Answer any *two* questions. Each carries 5 weightage.

15. Prove that the distribution function of a random variable X is non decreasing, continuous on the right with $F(-\infty) = 0$ and $F(\infty) = 1$. Conversely every function F with the above properties is the distribution function of a random variable on some probability space.
16. State and prove Monotone Convergence theorem.
17. a) Discuss the association between Liaponouv and Lindberg condition.

    b) State and prove Liaponouv CLT.
18. Discuss the properties of conditional expectation?

(2 × 5 = 10 weightage)