D2AST2301 (2 Pages) Name................................
Reg.No............................

SECOND SEMESTER M.Sc. DEGREE EXAMINATION, APRIL 2024
(Regular/Improvement/Supplementary)

STATISTICS
FMST2C06-ESTIMATION THEORY

Time: 3 Hours **Maximum Weightage: 30**

Part A: Answer any *four* questions. Each carries 2 weightage.

1. Define sufficiency. Let $X_1, X_2, \dots, X_n$ is a random sample from a population having pdf $f(x) = e^{-(x-\theta)},\ x > \theta$. Find a sufficient statistic for θ.
2. Define complete family of distributions. Is $N(0, \sigma^2)$ a complete family of distribution?
3. Describe method of percentile for finding consistent estimator.
4. Define consistency. Give an example to show that a consistent estimator need not be unbiased.
5. Explain method of moments. Find the moment estimator of θ in B$(1, \theta)$.
6. Explain the concept of shortest expected length confidence interval.
7. What do you mean by a pivot in confidence estimation? Give an example.

(4 × 2 = 8 weightage)

Part B: Answer any *four* questions. Each carries 3 weightage.

8. a) If T is a complete statistic show that any one to one function of T is also complete.

   b) Let T be a UMVUE of g(θ), and U be an unbiased estimator of *zero* then show that T is UMVUE if and only if Cov (T, U)= 0.

9. a) State and prove Cramer Rao inequality.

   b) Find CRLB for the variance of an unbiased estimator of θ in sampling from N$(\theta, 1)$.

10. Show that for distribution belonging to one parameter exponential family, the MLE is CAN for θ with asymptotic variance $\frac{1}{nI(\theta)}$.

11. a) Explain the method of selecting between consistent estimators.

    b) Show that $\frac{n\bar{x}}{n+1}$ is consistent for λ in the case of Poisson population P(λ).

12. Explain maximum likelihood estimation. If X follows hyper geometric distribution H(x: n, M, N), find the MLE of N if n and M are known.
13. Show that under certain regularity conditions the maximum likelihood estimator is consistent and asymptotically normal.

(P.T.O.)

14. Explain large sample confidence interval.

(4 × 3 = 12 weightage)

Part C: Answer any *two* questions. Each carries 5 weightage.

15. a) Let $X_1, X_2, \ldots, X_n$ is a random sample from $N(\mu, \sigma^2)$ population. Find sufficient statistic for: (i) μ when σ^2 is known; (ii) σ^2 when μ is known; (iii) μ and σ^2 when both are unknown.

    b) State and prove Basu's theorem.

16. a) State and prove Invariance property of CAN estimator.

    b) Examine the consistency of sample mean and sample median as an estimator of θ in the case of a Cauchy distribution $f_\theta(x) = \frac{1}{\pi}\frac{1}{1+(x-\theta)^2}, -\infty < x < \infty$.

17. a) Show that the Bayes estimator of θ under the absolute error loss function is the median of the posterior distribution.

    b) A random sample of size n is taken from Poisson (λ) population with λ has an exponential distribution with mean unity. Find the Bayes estimator of λ under squared error loss function.

18. a) Let $X_1, X_2, \ldots, X_n$ be a random sample from U (0,θ). Obtain an unbiased confidence interval for θ.

    b) Find a confidence interval with confidence coefficient (1-α) for the difference between means of two normal populations with common unknown variance.

(2 × 5 = 10 weightage)