(3 pages)

D2AMT2305

Name ....
Reg.No ....

SECOND SEMESTER M. Sc. DEGREE EXAMINATION, APRIL 2024
(Regular/Improvement/Supplementary)
MATHEMATICS
FMTH2C10 - OPERATIONS RESEARCH

Time: 3 Hours **Maximum Weightage: 30**

Part A : Answer *all* questions. Each carries 1 weightage.

1. **Show that for $X \in E_3$, $f(X) = ||X||$ is a convex function.**
2. **Explain the terms feasible solution, basic feasible solution and optimal solution with the help of examples.**
3. **Show that if an objective function $f(X)$ of an LP problem is obtained at more than one vertex of feasible region S_F, the minimum is attained at the convex linear combinations of these vertices also.**
4. **Explain the concept of dual of an LP problem.**
5. **Show that transportation problem has a triangular basis.**
6. **Show that if an optimal solution for a LP problem is a lower bound for the optimal solutions of the corresponding ILPP, provided the feasible region for the corresponding ILPP is non empty.**
7. **Find the strategic saddle point and value of the game for the following rectangular game .**

*	1	2
1	5	1
2	3	4

8. **Explain the notion of dominance.**

(8 x 1= 8 weightage)

Part B: Answer any *two questions from each unit.* Each carries 2 weightage.

UNIT I

9. **Show that every positive semi definite quadratic forms are convex function.**
10. **Show that the optimum value of the primal, if it exists, is equal to the optimum value of the dual.**

(P.T.O.)

11. Solve the given LPP :

$$\textbf{Maximize } 5x_1 - 3x_2 + 4x_3 \textbf{ ;}$$
$$\textbf{subject to } x_1 - x_2 \leq 1,\ -3x_1 + 2x_2 + 2x_3 \leq 1,\ 4x_1 - x_3 = 1,$$
$$x_2,\ x_3 \geq 0,\ x_1 \textbf{ unrestricted in sign.}$$

UNIT II

12. Solve by dual simplex method:

$$\textbf{Minimize } 2x_1 + 3x_2 \textbf{ ;}$$
$$\textbf{subject to } 2x_1 + 3x_2 \leq 30;\quad x_1 + 2x_2 \geq 10;\quad x_1,\ x_2 \geq 0.$$

13. Explain the Caterer problem and express it in the standard transportation form.

14. Find an optimal solution for the transportation problem:

*	D1	D2	D3	D4	D5	availability
O1	10	8	6	9	12	50
O2	5	3	8	4	10	90
O3	7	9	6	10	4	60
demand	100	80	70	40	20	.

UNIT III

15. Find the maximum flow in the graph with the following arcs and non negative arc capacities.

arc	(a,1)	(a,2)	(1,2)	(1,3)	(1,4)	(2,4)	(3,2)	(3,4)	(4,3)	(3,b)	(4,b)
arc capacity	8	10	3	4	2	8	3	4	2	10	9

16. Show that $E(\zeta_i, Y_0) \leq E(X_0, Y_0) \leq E(X_0, \eta_j)$, **where** ζ_i **and** η_j **represents the pure strategies.**

17. Minimize $-2x_, - 3x_2$ **subject to** $2x_1 + 2x_2 \leq 7;\ 0 \leq x_1 \leq 2;\ 1 \leq x_2 \leq 3;\ x_1,\ x_2$ **are integers.**

(6 x 2= 12 weightage)

Part C: Answer any *one* question. Each carries 5 weightage.

18. A student has 100 hours to prepare three subjects P, C and S. For every one hour of study he hopes to get 1 mark in P, 2 marks in C and 3 marks in S. He has already secured 60, 70 and 67 marks respectively in the course work. He must get at least 30, 30, and 33 more marks to pass in each of these subjects. Also he cannot get more than 100 marks in any written paper. Formulate the problem as a linear programming problem and maximize his total marks in the examination by managing the hours of study of each study.

19. Use branch and bound method to solve the ILPP
Maximize $3x_1 + 5x_2$
subject to $2x_1 + 4x_2 \leq 25$; $x_1 \leq 8$; $x_2 \leq 5$; x_1, x_2 **non negative integers.**

20. For the following LP problem,

$$\textbf{Maximize } f = x_1 - x_2 + 2x_3 \;; \quad \textbf{subject to}$$
$$x_1 - x_2 + x_3 \leq 4; \quad x_1 + x_2 - x_3 \leq 3; \; 2x_1 - 2x_2 + 3x_3 \leq 15;$$
$$x_1,\, x_2,\, x_3 \geq 0.$$

the optimal table is given below, where x_4, x_5, x_6 **are slack variables:**

Basis	Values	x_1	x_2	x_3	x_4	x_5	x_6
x_3	21	4	0	1	0	2	1
x_4	7	2	0	0	1	1	0
x_2	24	5	1	0	0	3	1
$-f$	18	2	0	0	0	1	1

Carry out the sensitivity analysis and find the change in the optimal solution if:

i) **the objective function is changed to** $3x_1 + x_2 + 5x_3$;

ii) **first constrained is deleted;**

iii) **coefficient of** x_1 **in the objective function changes to 2.**

21. State and Prove the fundamental theorem of rectangular games.

(2 x 5= 10 weightage)