

D2AMT2304

Reg.No.....

Name:

SECOND SEMESTER M.Sc. DEGREE EXAMINATION, APRIL 2024
(Regular/Improvement/Supplementary)
MATHEMATICS
FMTH2C09: ODE & CALCULAS OF VARIATIONS

Time : 3 Hours

Maximum Weightage: 30

Part A

Answer *all* questions. Each carries 1 weightage.

1. Find the normal form of the Bessel's equation $x^2y'' + xy' + (x^2 - p^2)y = 0$.
2. Locate and classify singular points of $x^2y'' + (2 - x)y' = 0$.
3. Give the confluent hypergeometric equation.
4. Give two linearly independent solutions of $y'' + y = 0$.
5. Explain simple critical point.
6. Determine whether $E(x, y) = 2x^2 + xy + 3y^2$ is positive definite, negative definite or neither.
7. Find the critical points of $\frac{d^2x}{dt^2} + \frac{dx}{dt} - (x^3 + x^2 - 2x) = 0$.
8. Give the hyper geometric series.

(8 × 1 = 8 weightage)

Part B

Answer any *two* questions from each unit. Each carries 2 weightage.

Unit 1

9. If $q(x) < 0$, and if $u(x)$ is a nontrivial solutions of $u'' + q(x)u = 0$, then prove that $u(x)$ has at most one zero.
10. Prove that the equation $x^2y'' - 3xy' + (4x + 4)y = 0$ has only one Frobenius series solution and find it.
11. Find the general solution of $(1 + x^2)y'' + 2xy' - 2y = 0$ in terms of power series in x .

(P.T.O.)

Unit 2

12. Find the exact solution of the I.V.P. $y' = 2x(1 + y)$, $y(0) = 1$. Starting with $y_0 = 1$, calculate $y_1(x)$, $y_2(x)$ and $y_3(x)$.
13. Find first three terms of the Legendre series of $f(x) = \begin{cases} 1 & \text{if } -1 \leq x < 0, \\ x & \text{if } 0 \leq x \leq 1 \end{cases}$
14. State and prove orthogonality properties of Bessel functions.

Unit 3

15. Find the general solution of the system $\begin{cases} \frac{dx}{dt} = x \\ \frac{dy}{dt} = -y \end{cases}$
16. If there exists a Liapunov function $E(x, y)$ for the system $\begin{cases} \frac{dx}{dt} = F(x, y) \\ \frac{dy}{dt} = G(x, y), \end{cases}$ then prove that the critical point $(0, 0)$ is stable.
17. Find the curve of fixed length L that joins the points $(0, 0)$ and $(1, 0)$, lies above the x - axis, and encloses the maximum area between itself and the x - axis.

(6 × 2 = 12 weightage)

Part C

Answer any two questions. Each carries 5 weightage.

18. Verify that the origin is a regular singular point and calculate two independent Frobenius series solutions of the $4xy'' + 2y' + y = 0$.
19. (a) Solve Bessel's equation near the origin.
(b) Prove that $f(x, y) = \sqrt{y}$ does not satisfy Lipschitz condition on the rectangle $|x| \leq 1$ and $0 \leq y \leq 1$.
20. (a) Determine the nature and stability properties of the critical point $(0, 0)$ for the autonomous system

$$\begin{cases} \frac{dx}{dt} = 2x \\ \frac{dy}{dt} = 3y \end{cases}$$

- (b) Show that $(0, 0)$ is a simple critical point of the system

$$\begin{cases} \frac{dx}{dt} = -2x + 3y + xy \\ \frac{dy}{dt} = -x + y - 2xy^2. \end{cases}$$

21. State and prove Picard's theorem.

(2 × 5 = 10 weightage)