D2AMT2303 **(2 Pages)** **Name ....**
Reg.No..........................

SECOND SEMESTER M.Sc. DEGREE EXAMINATION, APRIL 2024
(Regular/Improvement/Supplementary)

MATHEMATICS
FMTH2C08: TOPOLOGY

Time: 3 Hours **Maximum Weightage: 30**

Part A: Answer *all* questions. Each carries 1 weightage.

1. Define neighbourhood of a point in a topological space. Prove that a subset of a topological space is open iff it is a neighbourhood of each of its points.
2. Define base of a topological space. Illustrate with an example.
3. Define co-finite topology and prove that every cofinite space is compact.
4. When do we say that a topological property is divisible? Prove that being a finite space is divisible.
5. Define a local base at a point x in a space X. Give an example. When will you say that a space is first countable.
6. Prove that the topological product of any finite number of connected space is connected.
7. State Urysohn's lemma.
8. Prove that every compact Hausdorff space is a T_3 space.

(8 × 1 = 8 weightage)

Part B: Answer any *two* questions from each unit. Each carries 2 weightage.

Unit 1

9. Prove that the real line with the semi-open interval topology is not second countable.
10. Prove that in a second countable space every open cover of it has a countable subcover.
11. For a subset A of a space X, prove that $\overline{A} = A \cup A'$.

Unit 2

12. Let C be a collection of connected subsets of a space X such that no two members of C are mutually separated. Then prove that union of all elements of C is connected.
13. Prove that every open, subjective map is a quotient map.
14. Prove that every quotient space of a locally connected space is locally connected.

(P.T.O.)

Unit 3

15. Suppose *y* is an accumulation point of a subset A of a T_1 space X. Then prove that every neighbourhood of *y* contains infinitely many points of A.

16. Prove that all metric spaces are T_4.

17. Prove that every regular, Lindelof space is normal.

(6 × 2 = 12 weightage)

Part C: Answer any *two* questions. Each carries 5 weightage.

18. (a) Describe the convergence of sequences with respect to the cofinite topology on a set X.

    (b) Determine the topology induced by a discrete metric on a set.

19. (a) Prove that every continuous real-valued function on a compact space is bounded and attains its extrema.

    (b) Prove that every second countable space is first countable. Is the converse true? Justify your answer.

20. (a) In a topological space X, prove the following:

    i) Components are closed sets.

    ii) Any two distinct components are mutually disjoint.

    iii) Every non-empty connected subset is contained in a unique component.

    iv) Every space is the disjoint union of its components.

    (b) Prove that every open subset of the real line in the usual topology can be expressed as the union of mutually disjoint open intervals.

21. A be a closed subset of a normal space X and suppose f: A→ $[-1,1]$ is a continuous function. Then prove that there exists a continuous function F: X → [-1, 1] such that F(x)= f(x) for all x ∈A.

(2 × 5 = 10 weightage)