D2AST2203 (2 Pages) Name............................

Reg.No..........................

SECOND SEMESTER M.Sc. DEGREE EXAMINATION, APRIL 2023

(Regular/Improvement/Supplementary)

STATISTICS

FMST2C08- PROBABILITY THEORY

Time: 3 Hours **Maximum Weightage: 30**

Part A: Answer any *four* questions. Each carries 2 weightage.

1. State Slutsky's theorem. Write its applications.
2. Provide a definition of a random variable and demonstrate how the number of successes from a coin toss can be represented as a random variable.
3. State Lindberg-Levy and Liapunov Central Limit Theorem.
4. What are distribution functions and how are they related to induced probability functions?
5. State Martingale, super and sub martingale.
6. Define almost sure convergence. Write an example.
7. State and prove the relationship between characteristic functions and moments.

(4 × 2= 8 weightage)

Part B: Answer any *four* questions. Each carries 3 weightage.

8. State and prove Borel 0-1 Law.
9. Let X and Y be two random variables such that their joint distribution is given by the following probability density function:

$$\boldsymbol{f(x,y)}\begin{cases}\boldsymbol{4xy} & \boldsymbol{for}\ \boldsymbol{0} < x < 1, 0 < y < 1, and\ x < y \\ & \boldsymbol{0} \quad \boldsymbol{otherwise}\end{cases}$$

   Show that X^2 and Y^2 are independent, even though X and Y are dependent.

10. State and prove the Strong Law of Large Numbers.
11. Prove that convergence in probability implies convergence in distribution and determine whether the converse is true. If the converse is not true, provide the necessary assumptions and proof to demonstrate this result.
12. State and prove the Continuity Theorem for characteristic functions.
13. State and prove the Helly-Bray Theorem for characteristic functions.
14. State and prove the Doob decomposition theorem for martingales.

(4 × 3 = 12 weightage)

(P.T.O.)

Part C: Answer any *two* questions. Each carries 5 weightage.

15. State and prove the Inversion Theorem for characteristic functions.

16. a) State and prove the Scheffé's Lemma.
    b) State and prove the Weak Law of Large Numbers.
    c) A fair six-sided die is rolled repeatedly, and the average value of the rolls is recorded after each roll. What is the probability that the average value will be within 0.1 of its expected value after 100 rolls?

17. State and prove Radan-Nikodym theorem.

18. State and prove Kolmogorov 0-1 law.

($2 \times 5 = 10$ weightage)