D2AST2201 **(2 Pages)** **Name................................**

Reg.No............................

SECOND SEMESTER M.Sc. DEGREE EXAMINATION, APRIL 2023
(Regular/Improvement/Supplementary)

STATISTICS
FMST2C06-ESTIMATION THEORY

Time: 3 Hours **Maximum Weightage: 30**

Part A: Answer any *four* questions. Each carries 2 weightage.

1. State and prove Factorization theorem.
2. Define the method of moments estimation and use it to find an estimator for all the two parameters of a Binomial distribution.
3. Show that the MLE is consistent under certain conditions.
4. Define consistent asymptotically normal (CAN) estimators. Write an example.
5. State and prove the Basu's Theorem.
6. Derive the formula for the shortest expected length confidence interval for the mean of a normal distribution.
7. Define Invariance property. Explain about the Invariance property of consistent estimators.

(4 × 2 = 8 weightage)

Part B: Answer any *four* questions. Each carries 3 weightage.

8. State and prove the Lehmann-Scheffé Theorem.
9. Use the Bayesian method of estimation to find an estimator for the mean of a normal distribution with known variance.
10. Explain how to use the Delta Method to find the asymptotic distribution of a function of a CAN estimator.
11. Consider a random sample of size n from a uniform distribution on the interval $(0, \theta)$. Find a consistent estimator for θ using the method of percentiles.
12. Use the Cramer-Huzurbazar theorem to find the asymptotic distribution of the MLE for a one-parameter exponential family.
13. State and prove the Rao-Blackwell Theorem.
14. Explain how the method of moments can be used to determine consistent estimators.

(4 × 3 = 12 weightage)

(P.T.O.)

Part C: Answer any *two* questions. Each carries 5 weightage.

15. Define UMVUE. Let $X \sim N(\mu, \sigma^2)$ then Find UMVUE for

    a. μ

    b. σ

    c. α^{th}quantile.

16. State and prove Cramer-Huzurbazar theorem.

17. Use the central limit theorem to construct a large sample confidence interval for the mean of a population with known and unknown variance.

18. a) Construct a confidence interval for the variance of a normal distribution using the chi-squared distribution.
    b) Define an unbiased confidence interval and show how to construct one for the mean of a normal distribution.

(2 × 5 = 10 weightage)