SECOND SEMESTER M.Sc. DEGREE EXAMINATION, APRIL 2023
(Regular/Improvement/Supplementary)

PHYSICS
FPHY2C06: MATHEMATICAL PHYSICS II

Time: 3 Hours   **Maximum Weightage: 30**

Part A: Short answer questions. Answer *all* questions. Each carries 1 weightage.

1. Find A and B such that $f(z)=(x^2+Ay^2-2xy)+i(Bx^2-y^2+2xy)$ is analytic. Hence, find $f'(z)$.

2. Using Cauchy's integral formula, determine the value of $\int_C \frac{3z^2+z}{z^2-1} dz$.

3. State and explain rearrangement theorem.

4. Show that every element of an abelian group forms a class by itself.

5. Write a note on Lie groups.

6. Explain Rayleigh-Ritz variational technique.

7. What are the merits of using integral equations?

8. Obtain the eigen function expansion of Green's function.

(8 x 1= 8 weightage)

Part B: Essay questions. Answer any *two* questions. Each carries 5 weightage.

9. Derive the Laurent's expansion of a complex function. Obtain the Laurent's series of

   $f(z) = \frac{1}{(1+z^2)(z+2)}$ when $1 < |z| < 2$.

10. Show that the symmetry transformations of a square constitute a group. Derive its classes.

11. a) Derive Euler's equation by applying variational principle.

    b) Consider two parallel coaxial wire circles connected by a surface of minimum area that is generated by revolving a curve y(x) about the x-axis. Determine the curve y(x) such that the area of the resulting surface will be a minimum.

12. Obtain the one-dimensional Green's function of Sturm Liouville differential equation. Hence, list out the properties of a 3-D Green's function.

(2 × 5 = 10 weightage)

Part C: Problems. Answer any four questions. Each carries 3 weightage.

13. Use contour integration to evaluate the real integral $\int_0^\infty \frac{dx}{(1+x^2)^3}$.

14. Find the subgroups of: a) cyclic group and b) a non-cyclic group of order 4.

(P.T.O.)

15. Explain isomorphism and homomorphism with suitable examples.

16. Find the ratio of R (radius) to H (height) that will minimize the total surface area of a right circular cylinder of fixed volume.

17. Using Laplace transform method, solve $\phi(x) = x + \int_0^x (t - x)\,\phi(t)dt$.

18. Solve the equation $\phi(x) = 1 - 2\int_0^x t\,\phi(t)dt$.

19. Find the Green's function for the boundary value problem y"(x) − y(x) = f(x) with boundary conditions y(±∞) = 0.

(4 x 3 = 12 weightage)