D2AMT2204 (2 Pages) Name: ....

Reg.No.: ....

SECOND SEMESTER M.Sc. DEGREE EXAMINATION, APRIL 2023
(Regular/Improvement/Supplementary)
MATHEMATICS
FMTH2C09: ODE & CALCULAS OF VARIATIONS

Time: 3 Hours **Maximum Weightage: 30**

Part A : *Answer **all** questions. Each carries 1 weightage.*

1. Show that the zeros of the functions $a\sin x + b\cos x$ and $c\sin x + d\cos x$ are distinct and occur alternately whenever $ad - bc \neq 0$.
2. Determine the nature of the point at $x = 0$ for the equation $x^4 y'' + (\sin x)y = 0$.
3. Show that $xF(1, 1, 2; -x) = log(1 + x)$.
4. Show that $P_n(-x) = (-1)^n P_n(x)$, where $P_n(x)$ denotes the Legendre polynomial of degree n.
5. Show that $\frac{d}{dx}[x^n J_n(x)] = x^n J_{n-1}(x)$.
6. State Picard's local existence theorem.
7. Describe the phase portrait of the system
$$\begin{cases} \frac{dx}{dt} = 1, \\ \frac{dy}{dt} = 2. \end{cases}$$
8. Define extremals and stationary curve of Euler's differential equation.

(8x1= 8 weightage)

Part B : *Answer any **two** questions from each unit. Each carries 2 weightage.*

Unit I

9. State and prove Sturm Separation Theorem.
10. Obtain the power series solution of the equation $y'' + xy = 0$.
11. Find the general solution of the equation $(x^2 - x - 6)y'' + (5 + 3x)y' + y = 0$ near its singular point x = 3.

Unit II

12. State and prove the orthogonality property of Legendre polynomials.
13. Check whether $f(x, y) = x^2|y|$ satisfies a Lipshitz condition on the rectangle $|x| \leq 1$ and $|y| \leq 1$. Also check if $\frac{\partial f}{\partial y}$ exist at all points on this rectangle.

(P.T.O.)

14. Find the general solution of the following system:

$$\begin{cases} \frac{dx}{dt} = 5x + 2y, \\ \frac{dy}{dt} = -x + y. \end{cases}$$

Unit III

15. Determine the nature and stability properties of the critical point $(0,0)$ for the linear autonomous system:

$$\begin{cases} \frac{dx}{dt} = -2x, \\ \frac{dy}{dt} = 3y. \end{cases}$$

16. Verify that $(0,0)$ is a simple critical point for the system and determine its nature and stability properties:

$$\begin{cases} \frac{dx}{dt} = -x - y - 3x^2y, \\ \frac{dy}{dt} = -2x - 4y + y\sin x. \end{cases}$$

17. Find the stationary function of

$$\int_0^4 [xy' - (y')^2]dx$$

which is determined by the boundary conditions $y(0) = 0$ and $y(4) = 3$.

(6x2= 12 weightage)

Part C : *Answer any* ***two*** *questions. Each carries 5 weightage.*

18. Find the general solution of Gauss's Hypergeometric equation.
19. Find Frobenius series solution of the equation $2xy'' + (3 - x)y' - y = 0$.
20. Explain Picard's method of successive approximations. Find the exact solution of the initial value problem $y' = x+y$, $y(0) = 1$. Apply Picard's method to find its approximate solution (staring with $y_0(x) = 1$) and compare with the exact solution.
21. a) Determine whether the function $-x^2-4xy-5y^2$ is positive definite, negative definite, or neither.

    b) Obtain Euler's differential equation for an extremal.

(2x5= 10 weightage)