**D2AMT2203** **(2 Pages)** **Name......................**

**Reg.No...................**

**SECOND SEMESTER M.Sc. DEGREE EXAMINATION, APRIL 2023**

(Regular/Improvement/Supplementary)

**MATHEMATICS**

**FMTH2C08 - TOPOLOGY**

**Time: 3 Hours** **Maximum weightage: 30**

## Part A

**Answer *all* questions. Each carries 1 weightage.**

1. Prove that open surjective map is a quotient map.
2. Let $(X, \tau)$ be a topological space abd $\mathcal{B}$ is a subfamily of $\tau$. Then prove that $\mathcal{B}$ is a base for $\tau$ if and only if for any $x \in X$ and an open set $G$ containing $x$, there exists $B \in \mathcal{B}$ such that $x \in B$ and $B \subset G$.
3. Prove that every path connected space is connected.
4. Define projection functions. Prove that projection functions are open.
5. Let $f : X \to Y$ is a continuous function where X and Y are topological spaces. Prove that Graph of $f$ defined by $G = \{(x, f(x)) : x \in X\}$ is homeomorphic to X.
6. Prove that every co-finite space is compact.
7. Suppose $y$ is an accumulation point of a subset $A$ of a $T_1$ space X. Then prove that every nieghbourhood of $y$ contains infinitely many points of $A$.
8. Prove that product of Hausdroff spaces is Hausdroff.

**(8x1= 8 weightage)**

## Part B

**Answer any *two* questions from each unit. Each carries 2 weightage.**

### Unit 1

9. Define heriditary property. Prove that metrizability is a heriditary property.
10. Prove that a subset $A$ of a space X is dense in X if and only if, for every non-empty open subset $B$ of X, $A \cap B \neq \phi$. Justify $\mathbb{Q}$ is dense in $\mathbb{R}$.
11. For a subset A of a space $X$, Prove that
$\overline{A} = \{y \in X : \ every\ neighbourhood\ of\ y\ intersects\ A\ non-vacously\}$

**(P.T.O.)**

## Unit 2

12. Prove that every separable space satisfies the countable chain condition.

13. Prove that every quotient space of locally connected space is locally connected.

14. If $C$ is a connected subset of a space $X$, then prove that any set $D$ such that $C \subset D \subset \overline{C}$ is connected.

## Unit 3

15. Prove that a continuous bijection from a compact space onto a $T_2$ space is a homeomorphism.

16. Prove that every regular Lindeloff space is normal.

17. Let $X$ be a completely regular space. Let $F$ be a compact subset of $X$ and $C$ is a closed subset of $X$, and $F \cup C = \phi$. Then prove that there exists a continuous function from $X$ into unit interval which takes the value 0 at all points of $F$ and value 1 at all points of $C$.

**(6x2= 12 weightage)**

## Part C

**Answer any *two* questions. Each carries 5 weightage.**

18. Prove that the product topology on $\mathbb{R}^n$ coincides with the usual topology on it.

19. Prove that every closed and bounded interval is compact.

20. (a) Prove that all metric spaces are $T_4$.

    (b) Let $Y$ be a Hausdroff space. Prove that for any space $X$ and two continuous maps $f$ and $g$ from X to Y, the set $\{x \in X : f(x) = g(x)\}$ is closed in $X$.

21. State and prove Tietze extension theorem.

**(2x5= 10 weightage)**