D2AMT2205 **Name............................**

Reg. No. ....

SECOND SEMESTER M.Sc. DEGREE EXAMINATION, APRIL 2023
(Regular/Improvement/Supplementary)
MATHEMATICS
FMTH2C010: OPERATIONS RESEARCH

Time : 3 Hours **Maximum Weightage: 30**

Part A

Answer *all* questions. Each carries 1 weightage.

1. Show that the set $S \subseteq E_n$ of all points satisfying
$g_i(X) \leq 0,\ i = 1, 2, \cdots, m,\ X \in E_n$
is a convex set, where each $g_i(X)$ is a convex function.

2. Define a convex function. State and prove a necessary and sufficient condition for a differentiable function $f(X)$ defined over a convex domain $K \subseteq E_n$ to be convex.

3. Explain with an example, the necessity of artificial variables in a LPP.

4. Give the general form of a balanced transportation problem. Give the transportation matrix with 3 origins and 4 destinations.

5. Define the dual of an LP problem. Show that the dual of a dual is primal.

6. Discuss an algorithm to find a minimal spanning tree of a graph.

7. Define pure strategy, mixed strategy, saddle point, strategic saddle point and expectation of a rectangular game. Show that $E(\zeta_i, Y_0) \leq E(X_0, Y_0) \leq E(X_0, \eta_j)$, where ζ_i and η_j represents the pure strategies.

8. Find the strategic saddle point and value of the game for the following rectangular game.

*	1	2
1	4	0
2	2	3

(8 × 1 = 8 weightage)

(P.T.O.)

Part B

Answer any *two* questions from *each unit.* Each carries 2 weightage.

Unit I

9. Show that in LPP, if an objective function attains minimum at more than one of the vertices of the feasible region, then $f(X)$ is minimum at the convex linear combinations of these vertices.

10. What are simplex multipliers. Express the objective function $f(X)$ in terms of the non basic variables only using simplex multipliers.

11. Show that every positive linear combinations of convex functions in the convex set K is a convex function in K.

Unit II

12. Solve the dual and hence solve the given LPP :

$$\text{Maximize } y_1 + y_2 + y_3 \,;$$
$$\text{subject to } 2y_1 + y_2 + 2y_3 \leq 2;\;\; 4y_1 + 2y_2 + y_3 \leq 2; y_1,\, y_2,\, y_3 \geq 0.$$

13. Show that if the primal problem is feasible, then it has an unbounded optimum if and only if the dual is infeasible.

14. Find an optimal solution for the transportation problem

*	D1	D2	D3	availability
O1	4	5	7	10
O2	6	9	0	25
O3	3	5	1	25
O4	3	1	2	30
demand	20	20	15	.

Unit III

15. Minimize $-2\,x_{,} - 3\,x_2$ subject to
$2\,x_1 + 2\,x_2 \leq 7;\; 0 \leq x_1 \leq 2;\; 1 \leq x_2 \leq 3;\;\; x_1,\; x_2$ are integers.

16. Formulate the rectangular game as LPP for both players.

*	1	2	3
1	3	-4	2
2	1	-7	-3
3	-2	4	7

17. Tasks A, B, C, D, E, F, G, H, and I constitute a project. The notation $X \prec Y$ means that the task X must be finished before Y can begin. For this project it must be $A \prec D$, $A \prec E$, $B \prec F$, $D \prec F$, $C \prec G$, $C \prec H$, $F \prec I$, $G \prec I$. The time of completion of each task is as follows:

Task	A	B	C	D	E	F	G	H	I
Time	8	10	7	9	16	7	8	14	9

Draw the graph to represent the sequence of tasks and find the minimum time of completion of the project.

(6 x 2= 12 weightage)

Part C

Answer any *two* questions. Each carries 5 weightage.

18. Show that a basic feasible solution of the LP problem is a vertex of the convex set S_F of feasible solutions. Further more show that every vertex of S_F is a basic feasible solution.

19. a) Define triangular basis. Show that the transportation problem has a triangular basis.
b) Briefly describe the caterer problem and derive the problem in the standard transportation form.

20. Use branch and bound method to solve the ILPP
Maximize $3x_1 + 5x_2$
subject to $2x_1 + 4x_2 \leq 25$; $x_1 \leq 8$; $x_2 \leq 5$;
x_1, x_2 non negative integers.

21. Use the notion of dominance to solve the rectangular game

*	1	2	3	4
1	9	5	7	6
2	0	10	5	3

(2 x 5= 10 weightage)