D2APH2102 **(2 Pages)** **Name:..........................**
Reg. No:.......................

SECOND SEMESTER M.Sc. DEGREE EXAMINATION, APRIL 2022
(Regular/Improvement/Supplementary)

PHYSICS
FPHY2C06: MATHEMATICAL PHYSICS II

Time: 3 Hours **Maximum Weightage: 30**

Part A: Short answer questions. Answer *all* questions. Each carries 1 weightage.

1. Explain how Green's function acts as a propagator function.
2. Show that the group generated by two commuting elements A and B such that $A^2 = B^2 = E$ is cyclic. What is it's order?
3. What is the condition for the existence of the derivative of a complex function? The function *u(x,y)* and *v(x,y)* are the real and imaginary parts of an analytic function *w(z)*. Show that $\nabla^2 u = \nabla^2 v = 0$.
4. Discuss briefly the rules for the construction of character table.
5. Show that the function $y(x) = \frac{1}{(1+x^2)^{\frac{3}{2}}}$ is a solution of the Volterra integral equation $y(x) = \frac{1}{1+x^2} - \int_0^x \frac{t}{1+x^2} y(t)dt.$
6. Show that the integral $J = \int_{x1}^{x2} f(y, y_x, x)dx$ with $f = y(x)$ has no extreme values.
7. Determine the nature of singularity and evaluate the pole of the function $f(x) = \frac{1}{z^2+a^2}$.
8. Give the generators of the group C_{4v}.

(8 x 1= 8 weightage)

Part B: Essay questions. Answer any *two* questions. Each carries 5 weightage.

9. What are integral equations? Describe the Laplace transform method for solving an integral equation. The kernel of Volterra equation of the first kind $f(x) = \int_0^x k(x,t)\phi(t)dt$ has the form of *k(x-t)*. Assuming the required transforms exist, show that $\phi(x) = \frac{1}{2\pi i}\int_{r-i\infty}^{r+i\infty} \frac{F(s)}{K(s)} e^{xs} ds$.
10. Explain how Green's function can be used to solve a non-homogeneous differential equation. Solve the equation $y'' = f(x)$ using Green's function subject to the boundary condition $y(0) = 0; y'(1) = 0$.

(P.T.O.)

11. Find the most general form of 2 x 2 orthogonal matrix. Prove that the set of all 2 x 2 orthogonal matrix whose determinant is unity form a group. Obtain the generators for this group.
12. Describe briefly the concept of variation. Explain the Rayleigh Ritz variational techniques for the evaluation of eigen values.

(2 x 5 = 10 weightage)

Part C: Problems. Answer any four questions. Each carries 3 weightage.

13. Express Green's function in terms of it's eigen function for the differential equation of harmonic oscillator.
14. Develop a Laurent expansion of $f(z) = [z(z-1)]^{-1}$ about the point $z = 1$ valid for small values of $|z-1|$.
15. Prove that $\int_{-\infty}^{\infty} \frac{sin^2 x}{x^2} dx = \frac{\pi}{2}$.
16. Find the poles and residue of the function $z \cot z$.
17. Find the ratio of radius R to height H that will minimise the total surface area of a right circular cylinder of fixed volume.
18. Construct the symmetry group of an equilateral triangle. Show that C_{3v} is isomorphic to S_3.
19. Find the eigen values and corresponding eigen functions of the homogeneous Fredholm equation $\phi(x) = \lambda \int_{-1}^{1} (t + x)\, \phi(t) dt$.

(4 x 3 = 12 weightage)