

SECOND SEMESTER M.Sc. DEGREE EXAMINATION, APRIL 2022
(Regular/Improvement/Supplementary)

MATHEMATICS
FMTH2C10: OPERATIONS RESEARCH

Time: 3 Hours

Maximum Weightage: 30

Part A: Answer *all* questions. Each carries 1 weightage.

1. Show that the sum of two convex functions is again convex.
2. What is meant by degeneracy in LP?
3. Show that the set of feasible solutions is a convex set.
4. The dual of dual is the primal. Prove.
5. Prove that if an optimal solution of an LP problem is an integer or mixed integer vector then it is also an optimal solution of the corresponding MILP.
6. What is meant by unbalanced transportation problem?
7. Show that $\max_{\min} f(X, Y) \leq \min_{\max} f(X, Y)$ if they exist.
8. What is meant by sensitivity analysis?

(8 × 1 = 8 weightage)

Part B: Answer any *two* questions from each unit. Each carries 2 weightage.

Unit 1

9. Let $f(X)$ be defined in a convex domain $K \subseteq E_n$ and be differentiable. Then show that $f(X)$ is a convex function if and only if $f(X_1) - f(X_2) \geq (X_2 - X_1)' \nabla f(X_1)$ for all X_1, X_2 in K .
10. Show that a vertex of S_F is a basic feasible solution.
11. What is meant by simplex multipliers?

Unit 2

12. Show that the optimal value of the primal, if it exists, is equal to the optimal value of its dual.
13. Explain the important steps in formulating a LP problem with an example.

(P.T.O.)

14. Solve the transportation problem:

	D_1	D_2	D_3	
O_1	4	5	2	30
O_2	4	1	3	40
O_3	3	6	2	20
O_4	2	3	7	60
	40	50	60	

Unit 3

15. Give a general description of branch and bound method.

16. Find the maximum flow in the network described below.

Arc	$(a, 1)$	$(a, 2)$	$(1, 2)$	$(1, 3)$	$(1, 4)$	$(2, 4)$	$(3, 2)$	$(3, 4)$	$(4, 3)$	$(3, b)$	$(4, b)$
Capacity	8	10	3	4	2	8	3	4	2	10	9

17. State and prove the necessary and sufficient condition for the existence of saddle point of $f(X, Y)$.

(6 × 2 = 12 weightage)

Part C: Answer any two questions. Each carries 5 weightage.

18. Solve the following problem using simplex method. Also solve it by solving the dual graphically.

$$\text{Maximize } y_1 + y_2 + y_3$$

$$\text{subject to } 2y_1 + y_2 + 2y_3 \leq 2, 4y_1 + 2y_2 + y_3 \leq 2, y_j \geq 0, j = 1, 2, 3$$

19. Solve the following LP using cutting plane method.

$$\text{Minimise } 3x_1 - x_2$$

$$\text{subject to } -10x_1 + 6x_2 \leq 15, 14x_1 + 18x_2 \geq 63; x_1, x_2 \text{ are non-negative integers.}$$

20. Show that the maximum flow in a graph is equal to the minimum of the capacities of all possible cuts in it.

21. State and prove the fundamental theorem of rectangular games.

(2 × 5 = 10 weightage)