D2AMT2103 (2 Pages) Name............................

Reg.No..........................

SECOND SEMESTER M.Sc. DEGREE EXAMINATION, APRIL 2022
MATHEMATICS
FMTH2C08: TOPOLOGY

Time: 3 Hours **Maximum Weightage: 30**

Part A: Answer *all* questions. Each carries 1 weightage

1. If $\{A_\alpha\}$ is a family of topologies on X. Show that $\cap A_\alpha$ is a topology on X. Is $\cup A_\alpha$ a topology on X? Justify
2. Define usual topology and lower limit topology on $\mathbb{R}$. Establish a relation among them.
3. Prove that a subset of a topological space is open iff it is a neighbourhood of each of its points.
4. Prove that every continuous image of a compact space is compact.
5. Show that every cofinite space is compact.
6. Prove that if X is locally connected then components of open subsets of X are open in X.
7. Prove that in a Hausdorff space, limits of sequences are unique.
8. Prove that a continuous bijection from a compact space onto a Hausdorff space is a homeomorphism.

(8 × 1 = 8 weightage)

Part B: Answer any *two* questions from each unit. Each carries 2 weightage

Unit 1

9. Define a dense subset of a topological space. Prove that a subset A of a space is dense in X iff for every nonempty open subset B of X, $A \cap B \neq \phi$.
10. Prove that every open cover of a second countable space has a countable subcover.
11. Prove that $\overline{A} = A \cup A'$.

Unit 2

12. Prove that every second countable space is first countable.
13. Prove that a subset of $\mathbb{R}$ is connected iff it is an interval.
14. Prove that every quotient space of a locally connected space is locally connected.

Unit 3

15. Prove that all metric spaces are T_4.

16. Prove that every map from a compact space into a T_2 space is closed and the range of such a map is a quotient space of the domain.

17. Prove that every compact Hausdorff space is T_4.

(6 × 2 = 12 weightage)

Part C: Answer any *two* questions. Each carries 5 weightage

18. (1)Show that the product topology and the usual topology coincides on $\mathbb{R}^n$.

    (2)Define continuity on a topological space. Prove that the mapping f of topological space (X, τ) into an indiscrete space (Y, I), is continuous.

19. (1) Prove that metrisability is a hereditary property.

    (2) Prove that a discrete space is second countable if and only if the under lying set is countable.

20. (1)Show that every closed and bounded interval is compact.

    (2)Is the interval (0,1) compact? Justify your answer.

21. (1) Show that every regular, Lindeloff space is normal.

    (2)Prove that a compact subset in a Hausdorff space is closed.

(2 × 5 = 10 weightage)