D2AST2103 (3 Pages) Name............................

Reg.No..........................

SECOND SEMESTER M.Sc. DEGREE EXAMINATION, APRIL 2022

(Regular/Improvement/Supplementary)

STATISTICS

FMST2C08: PROBABILITY THEORY

Time: 3 Hours **Max. Weightage: 30**

Part A: Answer *any four* questions. Each carries *two* weightage

1. Define random variable. Illustrate it through a suitable example.
2. If X and Y are independent random variables and $g(.)$, a continuous real valued function, prove that $g(X)$ and $g(Y)$ are independent.
3. Define r^{th} mean convergence. Does r^{th} mean convergence imply convergence in probability. Justify.
4. State Kolmogorov's strong law of large numbers. Check whether a sequence of i.i.d Bernoulli's random variables obeys strong law of large numbers.
5. Define characteristic function. Is $\phi(t) = 1,\ t \in R$ a characteristic function? If so identify the random variable for which $\phi(t)$ is the characteristic function.
6. State Lindeberg-Levy form of central limit theorem.
7. Define martingale. If $\{X_n\}$ is a sequence of i.i.d random variables, check whether $Y_n = \prod_{i=1}^{n} X_i$ is a martingale.

(4×2= 8 weightage)

Part B: Answer *any four* questions. Each carries *three* weightage

8. **Let (Ω, F, P) be a probability space and $\{A_n\}$ be a monotone sequence of events on (Ω, F, P), prove that $\lim_{n\to\infty} P(A_n) = P(\lim_{n\to\infty} A_n)$.**

9. **What is a truncated random variable? What advantage you get if you truncate a random variable?**

10. **What are tail events? State and prove Kolmogorov's 0-1 law.**

11. **Show that characteristic function is uniformly continuous over the real line.**

12. **State and prove Liapunov's central limit theorem.**

13. **Define conditional expectation. Derive conditional probability from conditional expectation.**

14. **State and prove Borel-Cantelli lemma .**

(4×3= 12 weightage)

Part C: Answer *any two* questions. Each carries *five* weightage

15. **a) What do you mean by probability space induced by a random variable? If X is a Bernoulli random variable, what is its induced probability space?**

    b) If X and Y are two random variables with joint distribution function $F_{X,Y}(x, y)$, prove that X and Y are independent if and only if

$$F_{X,Y}(x, y) = F_X(x)F_Y(y), \quad x, y \in R.$$

16. **a) If $X_n \xrightarrow{P} X$, $Y_n \xrightarrow{P} Y$, α, β are real constants prove that $\alpha X_n + \beta Y_n \xrightarrow{P} \alpha X + \beta Y$.**

    b) State and prove Kolmogorov's inequality.

17. a) State and prove inversion theorem on characteristic function.

b) Find the probability density function corresponding to the characteristic function

$$\phi(t) = \begin{cases} 1 - |t| & \textbf{if } |t| \leq 1 \\ 0 & \textbf{otherwise.} \end{cases}$$

18. a) State and prove important properties of conditional expectation.

b) Write short notes on the following

(i) Doob decomposition theorem (ii) Stopping time.

(2×5= 10 weightage)