Name: ....

D2AST2001 Reg. No.: ....

SECOND SEMESTER M.Sc. DEGREE EXAMINATION, APRIL 2021
STATISTICS
FMST2C06: ESTIMATION THEORY

Time: 3 Hours **Maximum Weightage: 30**

Part - A: Answer any *four* questions. Each carries 2 weightage.

1. State and prove a sufficient condition for consistency.
2. Define a sufficient statistic. Explain its significance.
3. Define exponential family of distributions. Give an example.
4. Find Cramer-Rao lower bound for the unbiased estimate of θ in $N(\theta, 1)$.
5. Examine whether there is minimum variance bound estimate (MVBE) for θ in $f(x) = e^{-(x-\theta)}; \quad \theta \leq x < \infty, \theta > 0$.
6. Show that maximum likelihood estimate and moment estimate of the parameter θ in Poisson(θ) are the same.
7. Explain interval estimation. What will happen to the width of the confidence interval when the confidence level increases.

($4 \times 2 = 8$ weightage)

Part - B: Answer any *four* questions. Each carries 3 weightage.

8. Let X follows Poisson with parameter λ. Based on a single observation, find an unbiased estimator of (i) $\frac{1}{\lambda}$, (ii) $e^{-3\lambda}$, if one such exists.
9. Given a random sample of n independent observations on X with $f(x, \theta) = \frac{\theta}{x^{\theta+1}}, \; x > 1$ and $\theta > 0$. Find the MLE of θ and its asymptotic variance.
10. Find the Cramer-Rao lower bound for any unbiased estimator based on n independent observations for θ in $f(x) = \frac{1}{\pi[1+(x-\theta)^2]}; \quad -\infty < x, \theta < \infty$. Does there exist an unbiased estimator whose variance is equal to the bound obtained.
11. Given a random sample of n observations from Bernoulli with parameter θ distribution. Obtain the UMVUE of $\theta(1-\theta)$.
12. Let $X_1, X_2, ..., X_n$ be random sample from $N(\mu, \mu^2), \mu > 0$. Show that $(\sum x_i, \sum x_i^2)$ is sufficient for μ but not complete.
13. State and prove the Factorization theorem, and prove it for discrete case.

14. Let $X_1, X_2, ..., X_n$ be random sample from $N(\mu, \sigma^2)$, obtain an unbiased confidence interval for σ^2.

($4 \times 3 = 12$ weightage)

Part - C: Answer any *two* questions. Each carries 5 weightage.

15. (i) Define conjugate family. Show that gamma family is conjugate to the Poisson family.
(ii) A random sample of n observations are made on $X \sim G\left(1, \frac{1}{\theta}\right)$. Also, the prior distribution of θ is $\pi(\theta) = e^{-\theta}, \theta > 0$. Assume a squared error loss function and find the Bayes estimator of θ.

16. Let $X_1, X_2, ..., X_n$ be random sample of size n from a Poisson population with parameter λ.
(i) Obtain the UMVUE of $e^{-\lambda}$.
(ii) Find the variance of UMVUE of $e^{-\lambda}$.
(iii) Find the Cramer-Rao lower bound for the variance of the unbiased estimation of $e^{-\lambda}$.

17. (i) State and prove Rao-Blackwell theorem.
(ii) Establish Lehmann-Scheffe theorem.

18. (i) Define confidence interval. Explain the method of confidence interval using pivotal quantity.
(ii) Let $X_1, X_2, ..., X_n$ be random sample of size n from $N(\theta, \sigma^2 = 4)$. Obtain 95% confidence interval for $2\theta + 1$.

($2 \times 5 = 10$ weightage)