D2AST2001 (2 PAGES) Reg. No. ···············

Name:··················

SECOND SEMESTER M.Sc. DEGREE EXAMINATION, APRIL 2021

STATISTICS

FMST2C06: ESTIMATION THEORY

Time: Three Hours **Maximum Weightage: 30**

Part A: All questions can be answered. Each carries two weightage. (Ceiling 6 weightage).

1. Give an example of an ancillary statistics and a complete statistics.
2. What do you mean by a sufficient statistics? Obtain a sufficient statistics for σ^2 in $N(0, \sigma^2)$, $\sigma^2 > 0$.
3. What do you mean by BLUE? Give an example.
4. Examine whether sample median is a CAN estimator for μ in the Cauchy population $C(\mu, \sigma)$, $\mu \in R$, $\sigma > 0$.
5. What do you mean by an exponential family of densities? Give an example.
6. What do you mean by a pivotal quantity? Give an example.
7. Define Fisher information. Derive the Fisher information contained in a sample of size n from a Poisson distribution with mean λ.

Part B: All questions can be answered. Each carries four weightage. (Ceiling 12 weightage).

8. State and prove Cramer-Rao inequality.
9. Obtain UMVUE of $\mu^2 + 1$ in $N(\mu, 1)$, $\mu \in R$ based on a sample of size n.
10. Obtain the MLE and moment estimator of θ in $U(0, \theta)$, $\theta > 0$.
11. If T is a consistent estimator of θ and g is a continuous function, show that $g(T)$ is consistent for $g(\theta)$.
12. What do you mean by shortest confidence interval? Obtain the shortest confidence interval for μ in the case of normal population $N(\mu, \sigma^2)$, when both μ and σ^2 are unknown.

13. Distinguish between Bayesian and Fiducial confidence intervals.

14. State and prove Basu's theorem.

Part C: All questions can be answered. Each carries six weightage. (Ceiling 12 weightage).

15. Explain maximum likelihood method of estimation. Show that under some regularity conditions to be stated MLE is a CAN estimator.

16. a) State and prove Rao-Blackwell theorem.

    b) Obtain UMVUE of $1-e^{-\lambda}$, based on a random sample of size n from the Poisson population $\{P(\lambda),\ \lambda > 0\}$.

17. a) What do you mean by one parameter Cramer family? Give an example

    b) State and prove Cramer-Huzurbazar theorem.

18. a) What do you mean by a large sample confidence interval? Obtain confidence interval for the $p_1 - p_2$ based on random samples from two independent binomial populations $b(n, p_1)$ and $b(m, p_2)$

    b) Let X_1 and X_2 be two independent observations from the exponential population with pdf

    $$f(x;\theta) = e^{-(x-\theta)}, x > \theta.$$

    Let $Y = \min(X_1, X_2)$. Find the confidence coefficient of the interval $[Y - 1/2, Y + 1/2]$.