D2AMT2005 (2 Pages) Name............................

Reg.No..........................

SECOND SEMESTER M.Sc. DEGREE EXAMINATION, APRIL 2021

MATHEMATICS

FMTH2C10: OPERATIONS RESEARCH

Time: 3 Hours **Maximum Weightage: 30**

Part A: *All* questions can be answered. Each carries *one* weightage (Ceiling 6 weightage).

1. Define a convex function. Show that the function $f(x) = 2{x_1}^2 + 2{x_2}^2 + 4{x_3}^2 + 2\,x_1x_2 + 2\,x_1x_3 + 4\,x_3x_2$ is a convex function.
2. Explain the term basic feasible solution. Obtain an initial basic feasible solution for the LP problem: $Maximize\ f = 2\,x_1 + x_2 - x_3$, subject to the constraints $2\,x_1 - 5\,x_2 + 3x_3 \leq 4$, $3\,x_1 + 6x_2 - x_3 \geq 2$, $x_1 + x_2 + x_3 = 4$, for the non-negative variables x_1, x_2, x_3
3. Show that in LP problems if the optimal value of the objective function is assumed at more than one points of the feasible region, then the optimal value is attained at all those points which are the convex linear combinations of these points.
4. Explain the dual of a LP problem. Show that dual of a dual is primal.
5. Define the term 'triangular basis'. Show that a transportation problem has a triangular basis.
6. Define a tree. Show that a tree can have at the most only one centre.
7. What is meant by parametric linear programming.
8. Solve the rectangular game with pay-off matrix $\begin{bmatrix} 1 & 3 \\ -2 & 10 \end{bmatrix}$. Write the saddle points also.

Part B: *All* questions can be answered. Each carries *two* weightage (Ceiling of 12 weightage).

9. Show that a differentiable function f defined over a convex domain K is convex if and only if $f(X_2) - f(X_1) \geq (X_2 - X_1)'\,\nabla f(X_1),\ \forall\, X_1, X_2 \in K$
10. Show that any basic feasible solution of an LP problem is a vertex of the convex set of feasible solutions.
11. Solve the following LP problem: Minimize $f = 2\,x_1 - 3x_2 + 6x_3$, subject to the constraints $3\,x_1 - 4x_2 - 6x_3 \leq 2$, $2\,x_1 + x_2 + 2x_3 \geq 11$, $x_1 + 3\,x_2 - 2\,x_3 \leq 5$, for the non-negative variables x_1, x_2, x_3
12. a) Write the dual of Minimize $f = x_1 - 3x_2 - 2x_3$, subject to the constraints $2\,x_1 - 4x_2 \geq 12$, $3\,x_1 - x_2 + 2x_3 \leq 7$, $-4x_1 + 3\,x_2 + 8\,x_3 = 5$, for the non-negative variables x_1, x_2 and the unrestricted variable x_3

    b) Explain the complementary slackness condition.

(PTO)

13. Solve the following problem using dual simplex method: Minimize $f = 2x_1 + 3x_2$, subject to the constraints $2\,x_1 + 3x_2 \;\le 30,\;\; x_1 + 2x_2 \;\ge 10$, for the non-negative variables x_1, x_2

14. Define an unbalanced transportation problem. Solve

	D1	**D2**	**D3**	
O1	3	4	6	**100**
O2	7	3	8	**80**
O3	6	4	5	**90**
O4	7	5	2	**120**
	110	**110**	**60**	

15. Explain the cutting plane method in detail.

16. Schedule the 9 tasks named A , B, C, D, E, F, G, H, I so as to finish a project given below in minimum completion time. Find a critical path and minimum completion time. Scheduling has to be done according to the rule: A ~D, A ~ E, B ~ F, D ~ F, C ~ G, C ~ H, F ~ I, G ~ I, where X ~ Y means task X has to be finished before task Y can begin. The completion time in hours for each task is as follows.

Task	A	B	C	D	E	F	G	H	I
Time	8	10	7	9	16	7	8	14	9

17. Solve graphically Maximize $f = 4x_1 + 8x_2$, subject to the constraints $x_1 + 2x_2 \;\ge 20,\;\; 2x_1 + 2x_2 \;\le 100,\;\; x_1 - 3x_2 \;\le 0,\;\; 4x_1 - x_2 \;\ge 0$ for the non-negative variables x_1, x_2 . Analyze graphically how the optimal solution is affected if the objective function is replaced by $8x_1 + 4x_2$

Part C: *All* questions can be answered. Each carries *six* weightage (Ceiling 12 weightage).

18. State and establish a necessary condition for the existence of a saddle point for a real valued function $f(X,Y)$ with $max_X\, min_Y\; f(X,Y)$ and $min_Y\, max_X\; f(X,Y)$exists.

19. By the branch and bound method Minimize $f = -2x_1 - 3x_2$, subject to the constraints $2\,x_1 + 2x_2 \;\le 7,\;\; x_1 \le 2,\;\; x_2 \;\le 2$, for the non-negative integers x_1, x_2.

20. Define a loop in a transportation array. Describe the procedure for testing the optimality of a basic feasible solution in a transportation problem.

21. Solve the dual and hence solve the primal : Maximize $f = x_1 + x_2 + x_3$, subject to the constraints $2\,x_1 + x_2 + 2x_3 \;\le 2$, $4\,x_1 + 2x_2 + x_3 \;\le 2$,for the non-negative variables x_1, x_2 and x_3.