D2AMT2004 **(2 Pages)** **Name:......................**

Reg.No:....................

SECOND SEMESTER M.Sc. DEGREE EXAMINATION, APRIL 2021

MATHEMATICS

FMTH2C09: ODE & CALCULUS OF VARIATIONS

Time: 3 Hours **Maximum Weightage: 30**

Part A: All questions can be answered. Each carries one weightage. (Ceiling 6 weightage).

1 Find the singular points of the differential equation $(1-x^2)y'' - 2xy' + 5y = 0$.

2 State Sturm comparison theorem.

3 Write the hypergeometric series represented by $F[1, 2, 2, x]$.

4 Write Rodrigue's formula to find n^{th} Legendre polynomial $P_n(x)$.

Find $P_0(x)$,$P_1(x)$ and $P_2(x)$.

5 Express $J_2(x)$ in terms of $J_0(x)$ and $J_1(x)$.

6 Find the auxiliary equation of the autonomous system $\frac{dx}{dt} = 3x - 4y$, $\frac{dy}{dt} = x - y$.

7 Define a stable critical point.

8 Check whether the function $x^2 + 4xy + 5y^2$ is positive definite, negative definite or neither.

Part B: All questions can be answered. Each carries two weightage. (Ceiling of 12 weightage).

9 If $u(x)$ is any non trivial solution of $u'' + q(x)u = 0$ where $q(x) > 0$ for all $x > 0$ and if $\int_1^\infty q(x)dx = \infty$, prove that $u(x)$ has infinitely many zeros on the positive x-axis.

10 Find two independent Frobenius series solutions of the differential equation $4xy'' + 2y' + y = 0$.

11 Transform the differential equation $(x^2-1)y'' + (5x+4)y' + 4y = 0$ as a hypergeometric equation and hence find the general solution near $x = -1$.

12 Show that $J_{1/2}(x) = \sqrt{\frac{2}{\pi x}} \sin x$.

13 Solve the linear system $\frac{dx}{dt} = 3x - 4y$, $\frac{dy}{dt} = x - y$.

14 If $f(x, y)$ is any continuous function that satisfies Lipschitz condition on the strip $a \leq x \leq b$ and $-\infty < y < \infty$ and if $f(x_0, y_0)$ is any point of the strip, prove that the initial value problem $y' = f(x, y)$, $y(x_0) = y_0$ has one and only one solution on the interval $a \leq x \leq b$.

15 Show that $(0, 0)$ is an asymptotically stable critical point for the autonomous system $\frac{dx}{dt} = -3x^3 - y$, $\frac{dy}{dt} = x^5 - 2y^3$.

16 Let m_1 and m_2 be the roots of the indicial equation of the linear system $\frac{dx}{dt} = a_1x + b_1y$, $\frac{dy}{dt} = a_2x + b_2y$. If m_1 and m_2 are real, distinct and of the same sign, show that the critical point $(0, 0)$ is a node.

17 Find the curve joining the points (x_1, y_1) and (x_2, y_2) that yields a surface of revolution of minimum area when revolved about the x-axis.

Part C: All questions can be answered. Each carries six weightage.

(Ceiling 12 weightage).

18 State and prove Picard's theorem.

19 State and prove the orthogonal property of Legendre polynomials.

20 Show that Gauss hypergeometric equation has three regular singular points.

21 (a) Discuss different types of critical points of an autonomous system.

(b) Find the general solution of the linear system $\frac{dx}{dt} = -x$, $\frac{dy}{dt} = -2y$.

Find the differential equation of the paths and discuss the stability of the critical point.