D2AMT2003 (2 Pages) Name................................
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SECOND SEMESTER M. Sc. DEGREE EXAMINATION, APRIL 2021

MATHEMATICS

FMTH2C08: TOPOLOGY

Time : 3 Hours **Maximum Weightage: 30**

Part A: All questions can be answered. Each carries one weightage.

(Ceiling 6 weightage).

1. Define metrisable topological space and give an example of a space which is not metrisable.
2. Is $\overline{A \cap B} = \overline{A} \cap \overline{B}$. Justify your claim.
3. Find the interior of the set of integers, $\mathbb{Z}$ in $\mathbb{R}$ with usual topology.
4. Define boundary of a set and find the boundary of $\{2, 3, 5\}$ in $\mathbb{R}$ with discrete topology.
5. Define quotient map and give an example.
6. Show that the continuous image of a compact space is compact.
7. Give an example of a Hausdorff space which is not regular.
8. Define saturated sets.

Part B: All questions can be answered. Each carries two weightage.

(Ceiling of 12 weightage)

9. In an infinite cofinite topological space, show that a sequence is convergent if and only if there is at most one term of it which repeats infinitely often.
10. Let X be a set and $\mathcal{B}$ a family of its subsets covering X. If any $B_1, B_2 \in \mathcal{B}$ and $x \in B_1 \cap B_2$, there exists $B_3 \in \mathcal{B}$ such that $x \in B_3 \subset B_1 \cap B_2$, then show that there exists a topology on X with $\mathcal{B}$ as a base.
11. Show that the identity function $id_X : (X, \tau_1) \to (X, \tau_2)$ is continuous if and only if τ_1 is stronger than τ_2.
12. Show that the product topology is the weak topology determined by the projection functions.

13. Prove that every second countable space is separable.

14. Show that every quotient space of a locally connected space is locally connected.

15. Show that every continuous, one-to-one function from a compact space into a Hausdorff space is an embedding.

16. State and prove Wallace theorem.

17. Let $f : X \to [0, 1]$ be continuous. For each $t \in \mathbb{R}$, let $F_t = \{x \in X, f(x) < t\}$. Show that $f(x) = \inf \{t \in \mathbb{Q} : x \in F_t\}$.

Part C: All questions can be answered. Each carries six weightage.

(Ceiling 12 weightage).

18. (a). Let (X, τ) be a topological space and $\mathcal{S}$ a family of subsets of X. Show that $\mathcal{S}$ is a sub-base for τ if and only if $\mathcal{S}$ generates τ.

    (b). Show that $\overline{A} = A \cup A'$, where A is a subset of a topological space X.

19 (a). Prove that Metrisability is a hereditary property.

    (b). Prove that a subset of $\mathbb{R}$ is connected if and only if it is an interval.

20 (a). Show that every separable space satisfies the countable chain condition.

    (b). Show that a subset C is a path-component of a space X if and oly if C is a maximal subset w.r.t. the property of being path-connected.

21. (a). Prove that every regular, Lindelöff space is normal.

    (b). If every continuous real valued function on a closed subset A of a topological space X has a continuous extension to X, then show that X is normal.