

## SECOND SEMESTER M.Sc. DEGREE EXAMINATION, APRIL 2021

(Improvement/Supplementary)

## MATHEMATICS

## FMTH2C10: OPERATIONS RESEARCH

Time: 3 Hours

Maximum Weightage: 30.

**Part A: Answer all questions. Each question carries 1 weightage.**

1. Is the function  $f(x) = x^3$ ,  $x \in R$  a convex function? Justify your claim.
2. What is the canonical form of equations in linear programming problem?
3. Prove that the dual of the dual of a linear programming problem is primal.
4. What are simplex multipliers in a linear programming problem?
5. What is meant by degeneracy in transportation problem?
6. Prove that the maximum flow in a graph is equal to the minimum of the capacities of possible cuts in it.
7. What is Caterer Problem in Operation Research?
8. Describe a two-person zero – sum game.

**(8 x 1 = 8 Weightage)****Part B: Answer any two questions from each unit. Each question carries 2 weightage.****UNIT I**

9. State and prove necessary and sufficient conditions for a differentiable function  $f(x)$  defined in a convex domain to be a convex function.
10. Prove that a basic feasible solution of a linear programming problem is a vertex of the convex set of feasible solutions.
11. How do you recognise that a linear programming problem is unbounded while using the simplex method?

**UNIT II**

12. Prove that the optimum value of the primal, if it exists, is equal to the optimum value of the dual.
13. Prove that the value of the objective function, for any feasible solution of the primal is not less than the value of the objective function for any feasible solution of the dual.
14. Prove that the transportation array has a triangular basis.

**(PTO)**

### UNIT III

15. Describe the generalised problem of maximum flow.
16. Explain whether an integer programming problem can be solved by rounding off the corresponding simplex solution.
17. State and prove fundamental theorem of rectangular games.

(6 x 2 = 12 Weightage)

#### PART C: Answer any two questions. Each question carries 5 weightage.

18. a) Why do we introduce new variables in linear programming problem?
- b) Minimize  $f(x) = 2x_1 - 3x_2 + 4x_3$  subject to constraints  $3x_1 - 4x_2 - 6x_3 \leq 2$ ;  
 $2x_1 + x_2 + 2x_3 \geq 11$ ,  $x_1 + x_2 - x_3 \leq 5$ ;  $x_1, x_2, x_3 \geq 0$ .
19. (a) What is meant by loops in transportation array?
- (b) Solve the transportation problem for minimum cost with cost coefficients demands and supplies as in the following table.

	$D_1$	$D_2$	$D_3$	
$O_1$	4	5	2	30
$O_2$	4	1	3	40
$O_3$	3	6	2	20
$O_4$	2	3	7	60
	40	50	60	

20. Solve the integer linear programming problem: Maximize  $f(x) = 3x_1 + 4x_2$ . Subject to  
 $2x_1 + 4x_2 \leq 13$ ,  
 $-2x_1 + x_2 \leq 2$ ,  $2x_1 + 2x_2 \geq 1$ ,  $6x_1 - 4x_2 \leq 15$ ,  $x_1, x_2 \geq 0$  and  $x_1$  and  $x_2$  and integers.

21. (a). Solve graphically the game whose pay-off matrix is  $\begin{bmatrix} 2 & 7 \\ 3 & 5 \\ 11 & 2 \end{bmatrix}$
- (b). Describe the notion of dominance in game theory. .

(2 x 5 = 10 Weightage)

