D2AMT1903 (S1) **(2 Pages)** **Name: ……..…………**

Reg. No:……………....

SECOND SEMESTR M.Sc. DEGREE EXAMINATION, APRIL, 2021

(Improvement/Supplementary)

MATHEMATICS

FMTH2C08: TOPOLOGY

Time: 3 Hours **Maximum Weightage: 30**

PART A. Answer *all* questions. Each question carries 1 weightage.

1. Define convergence of a sequence in a topological space. Is limit of a sequence unique in a general topological space?
2. Define semi- open interval topology on **R** and compare it with the usual topology on **R.**
3. If X = { a,b,c }, τ = { ϕ, X, {a}, {a,b},{a,c} } and if A = { b,c}, find $\bar{A}$, the closure of A.
4. Define accumulation point of a subset of a topological space. Under what condition, every open set containing the accumulation point contains infinitely many points of the set?
5. If X is compact and f: X →Y is continuous and onto, show that Y is compact.
6. Define connected set. Under what condition will the union of connected sets be connected?
7. Define component of a topological space and show that components are closed sets.
8. Define path connected space. Is every connected space be path connected? Justify your claim.

(8 x 1 = 8 Weightage)

PART B. Answer any *two* questions from each unit. Each question carries 2 weightage.

UNIT – I

9. If (X, τ) is a topological space and $\mathcal{B} \subset \tau$, prove that $\mathcal{B}$ is a base for τ if and only if for any x ∈ X and any open set G containing x , there exists B ∈ $\mathcal{B}$ such that x ∈ B and B ⊂ G.
10. For a subset A of a topological space X, prove that $\bar{A}$ = { y ∈ X : every neighbourhood of y meets A non-vacuously }.
11. If (X, τ), (Y,σ) are topological spaces and f : X →Y is a function, show that f is continuous if and only if for all V ∈ σ , f^{-1} (V) ∈ τ.

UNIT – II

12. Show that the product topology is the weak topology determined by the projection functions.
13. Prove that every second countable space is separable.
14. Show that every continuous real valued function on a compact space is bounded.

(PTO)

UNIT – III

15. Define Hausdorff space. Show that in Hausdorff space, limits of sequences are unique.
16. Show that every completely regular space is regular.
17. Show that every regular, Lindeloff space is normal.

(6 x 2 = 12 Weightage)

PART C. Answer any *two* questions. Each question carries 5 weightage.

18. (a) Show that metrisability is a hereditary property.

    (b) If f: $X \to Y$ is continuous at a point $x_0 \in X$, prove that whenever a sequence $\{ x_n \}$ converges to x_0 in X, the sequence $\{ f(x_n) \}$ converges to $f(x_0)$ in Y.

19. (a) Define quotient map and show that every open surjective map is a quotient map.

    (b) Prove that every quotient space of a locally connected space is locally connected.

20. (a) Show that a topological space is T_1 if and only if for any $x \in X$, the singleton set $\{x\}$ is closed.

    (b) Show that all metric spaces are T_4.

21. Show that a topological space X is normal if and only if it has the property that for every two mutually disjoint, closed subsets A, B of X, there exists a continuous function $f : X \to [0,1]$ such that $f(x) = 0$ for all $x \in A$ and $f(x) = 1$ for all $x \in B$.

(2 x 5 = 10 Weightage)