D2AMT1902 (S1) (2 PAGES) **Name............................**
Reg.No...........................

SECOND SEMESTER M.Sc. DEGREE EXAMINATION, APRIL 2021

(Improvement/Supplementary)

MATHEMATICS

FMTH2C07: REAL ANALYSIS II

Time: 2 ½ Hours **Maximum Weightage: 30**

Part A: Answer *all* questions. Each carries 1 weightage

1. Prove that the translate of a measurable set is measurable.
2. Show that the Lebesgue measure is countably additive.
3. Prove that a real valued function that is continuous on its measurable domain is measurable.
4. Define Riemann integral of a function
5. Give an example of a function which is Lebesgue integrable, but, not Riemann integrable. Justify your answer.
6. Let f be a bounded measurable functions defined on a set of finite measure E. Then prove that $\left|\int_E f\right| \leq \int_E |f|$
7. Define bounded variation of a function.
8. Define a convex function. Give an example.

(8 × 1 = 8 Weightage)

Part B: Answer any *two* questions from each unit. Each carries 2 weightage.

Unit 1

9. Prove that the union of finite collection of measurable sets is measurable
10. State and prove Borel-Cantelli Lemma
11. Let the function f have a measurable domain E. Show that the following statements are equivalent
    a) $\forall\alpha, \{x: f(x) > \alpha\}$ is measurable.
    b) $\forall\alpha, \{x: f(x) \geq \alpha\}$ is measurable.
    c) $\forall\alpha, \{x: f(x) < \alpha\}$ is measurable.
    d) $\forall\alpha, \{x: f(x) \leq \alpha\}$ is measurable.

(PTO)

Unit 2

12. State and prove Chebychev's inequality.
13. Let f and g be bounded measurable functions defined on a set of finite measure E. Then prove that

(i) For any α and β

$$\int_E (\alpha f + \beta g) = \alpha \int_E f + \beta \int_E g$$

(ii) If $f \leq g$ on E, then $\int_E f \leq \int_E g$

14. Let the sequence of functions $\{f_n\}$ be uniformly integrable over E. If $\{f_n\} \rightarrow f$ pointwise a.e on E, then show that f is integrable over E.

Unit 3

15. Let C be a countable subset of the open interval (a, b). Then show that there is an increasing function on (a, b) that is continuous only at points in$(a. b)\sim C$.
16. State and prove Jordan's theorem.
17. Let E be a measurable set and $1 \leq p \leq \infty$. Suppose $\{f_n\}$ is a sequence in $L^p(E)$ that converges point wise a.e on E to the function f which belongs to $L^p(E)$. Then show that $\{f_n\} \rightarrow f$ in $L^p(E)$ iff $\lim_{n\rightarrow\infty} \int_E |f_n|^p = \int_E |f|^p$.

(6 × 2 = 12 Weightage)

Part C: Answer any *two* questions. Each carries 5 weightage

18. a) Let E be a measurable set of finite outer measure. Then show that for each $\varepsilon > 0$, there is a finite disjoint collection of open intervals $\{I_k\}_{k=1}^{n}$ for which if $O = \bigcup_{k=1}^{n} I_k$, then $m^*(E\sim O) + m^*(O\sim E) < \varepsilon$

    b) Show that the Lebesgue measure is countably additive.

19. State and prove Monotone Convergence theorem.
20. If the function f is monotone on the open interval (a, b), show that f is differentiable almost everywhere on (a, b).
21. Let the function f be continuous on the closed, bounded interval $[a, b]$. Then show that f is absolutely continuous on $[a, b]$ if the family of divided difference functions $\{Diff_h f\}_{0<h\leq 1}$ is uniformly integrable on $[a, b]$

(2 × 5 = 10 Weightage)