

FIRST SEMESTER M.Sc. DEGREE EXAMINATION, NOVEMBER 2025
(Regular/Improvement/Supplementary)

STATISTICS
FMST1C05- DISTRIBUTION THEORY

Time: 3 Hours

Maximum Weightage: 30

Part A: Answer any four questions. Each carries two weightage.

1. Define probability generating function of an integer valued random variable. If X_1, X_2, \dots, X_n are i.i.d rv's with PGF $P(S)$. Obtain the PGF of $\sum_{i=1}^n X_i$.
2. If $X \sim P(\lambda_1)$, $Y \sim P(\lambda_2)$, and are independent, find the conditional distribution of X given $X + Y$.
3. Define mixture distributions.
4. Define lognormal distribution. Find its r^{th} order raw moment.
5. If X_1, X_2, \dots, X_n are i.i.d random variables following the normal distribution, show that the sample mean \bar{X} is normally distributed.
6. Let $X_{(1)}, X_{(2)} \dots \dots X_{(n)}$ be the set of order statistics of independent random variables with common PDF $f(x) = \theta e^{-\theta x}$, $x \geq 0$. Find the distribution of r^{th} order statistics.
7. If X is distributed as $F(m, n)$, obtain the distribution of $Y = \frac{1}{X}$.

(4 × 2 = 8 weightage)

Part B: Answer any four questions. Each carries three weightage.

8. Let X and Y be independent random variables following the negative binomial distributions, $NB(r_1, p)$ and $NB(r_2, p)$ respectively. Show that the conditional probability mass function of X given $X + Y = t$ is hypergeometric.
9. Derive the recurrence relation satisfied by the cumulants of power series family of distribution.
10. Let X follows $N(0,1)$ and Y follows $N(0,1)$ be independent random variables. Find the distribution of X/Y and identify it.
11. Let $f(x)$ be the p.d.f of a random variable X . Find the distribution function and p.d.f of $Y = X^2$ and $Z = |X|$.
12. Consider the bivariate p.d.f

$$f(x, y) = \begin{cases} c, & 0 < x < y < 1 \\ 0, & \text{otherwise.} \end{cases}$$

Evaluate c . Obtain the marginal distributions of X and Y . Examine whether X and Y are independent.

13. State and prove the reproductive property of chi-square distribution.
14. If X_1 and X_2 are i.i.d random variables with density $f(x) = e^{-x}$, $x > 0$, show that $Z = \frac{X_1}{X_2}$ has an F distribution.

(4 × 3 = 12 weightage)

(P.T.O.)

Part C: Answer any two questions. Each carries five weightage.

15. Define the multinomial distribution. Obtain its characteristic function. Find $E(X_j)$, $V(X_j)$ and $\text{Cov}(X_j, X_l)$, $j \neq l$, where X_j and X_l are the j^{th} and l^{th} components of the multinomial variable. Show that the marginal distribution of any subset of variates associated with a multinomial distribution is also multinomial.
16. Show that for the Pearsonian distribution specified by $\frac{df(x)}{dx} = \frac{xf(x)}{b_0 + b_1x + b_2x^2}$, the types are determined by the nature of the roots of $b_0 + b_1x + b_2x^2 = 0$. Identify any two members of the family stating the condition.
17. Let $X_i \sim G(\alpha_i, \beta)$, $i = 1, 2$ and be independent. Show that the variates $Y_1 = X_1 + X_2$ and $Y_2 = \frac{X_1}{X_2}$ are independent and that $Y_1 \sim G(\alpha_1 + \alpha_2, \beta)$ and $Y_2 \sim \beta_2(\alpha_1, \alpha_2)$.
18. Define non-central t statistic. Derive the probability distribution of this statistics.

(2 × 5 = 10 weightage)