D1AST2503 **(2 pages)** **Name.....**
Reg.No.....

FIRST SEMESTER M.Sc. DEGREE EXAMINATION, NOVEMBER 2025
(Regular/Improvement/Supplementary)

STATISTICS
FMST1C03- ANALYTICAL TOOL FOR STATISTICS II

Time: 3 Hours **Maximum Weightage: 30**

Part A: Answer any *four* questions. Each carries *two* weightage.

1. Define basis and dimension of a vector space. Give an example of an infinite dimensional vector space.
2. Show that every subspace of a vector space has a complement.
3. Define row and column space of a matrix. What is the connection between the dimension of row and column space of a matrix?
4. Show that pre-multiplication by a non-singular matrix does not alter the rank of a matrix.
5. Explain the singular value decomposition of a matrix.
6. Show that the characteristic roots of an idempotent matrix are either 0 or 1.
7. If A is a positive definite matrix, then show that $|A| > 0$.

(4 × 2 = 8 weightage)

Part B: Answer any *four* questions. Each carries *three* weightage.

8. Show that all bases for a finite dimensional vector space have the same number of vectors.
9. Let A be a non-singular matrix of order n partitioned as $A = \begin{bmatrix} C_1 & C_2 \\ 0 & C_3 \end{bmatrix}$, where C_1 is a non-singular matrix of order $k < n$. Derive the inverse of A.
10. If A is an idempotent matrix, then show that rank(A) = trace(A).
11. State and prove Cayley-Hamilton Theorem. Mention one of its uses.
12. Explain the spectral decomposition of a real symmetric matrix. Discuss its properties.
13. State and prove rank nullity theorem.
14. If $\bar{A}$ is a generalized inverse of A, then show that the general solution of $AX = Y$ can be expressed in the form $X = \bar{A}Y + (H - I)z$, where $H = \bar{A}A$ and z is an arbitrary vector.

(4 × 3 = 12 weightage)

(P.T.O.)

Part C: Answer any *two* questions. Each carries *five* weightage.

15. Define an orthonormal basis and show that an orthonormal basis always exists. Construct an orthonormal basis from {(1, 1, 1), (-1, 0, -1), (-1, 2, 3)}.

16. If A and B are two square matrices of order n, then show that:

$$rank(A) + rank(B) - n \leq rank(AB) \leq Min\{rank(A), rank(B)\}$$

17. Define algebraic multiplicity and geometric multiplicity of a characteristic root. Show that geometric multiplicity of a characteristic root cannot exceed its algebraic multiplicity.

18. Reduce the quadratic form $x^2 + 2y^2 + 3z^2 + 2xy + 2yz - 2xz$ to its canonical form. Also identify the nature of the quadratic form.

(2 × 5 = 10 weightage)