

FIRST SEMESTER M.Sc. DEGREE EXAMINATION, NOVEMBER 2025
(Regular/Improvement/Supplementary)

STATISTICS
FMST1C02- ANALYTICAL TOOL FOR STATISTICS I

Time: 3 Hours

Maximum Weightage: 30

Part A: Answer any four questions. Each carries two weightage.

1. Define limit point of a set and give an example of a set with exactly one limit point.
2. Define: i) Perfect set; (ii) Connected sets.
3. Discuss the differentiability at $x=0$ of the function f , where $f: R \rightarrow R$ given by $f(x) = x^{1/3}$.
4. Define the Riemann-Stieltjes integral of $f(x)$ with respect to $\alpha(x)$.
5. If $f(x)$ is Riemann-Stieltjes integral with respect to $\alpha(x)$ ($f \in R(\alpha)$), on $[a,b]$, show that $f^2 \in R(\alpha)$.
6. Define: i) limit, ii) continuity and iii) differentiability of a function.
7. Define point wise and uniform convergence of a sequence of functions.

(4 × 2 = 8 weightage)

Part B: Answer any four questions. Each carries three weightage.

8. Define a metric space. For $x, y \in R$, if $d(x, y) = |x^2 - y^2|$, verify whether (R, d) is a metric space.
9. If (S, d) is a compact metric space and if the function $f: S \rightarrow R$ is continuous, then show that $f(S)$ is compact in R .
10. Find: i) $\log_{x \rightarrow \infty} (1 + \frac{1}{x})^x$ ii) $\lim_{x \rightarrow 0} x \sin(\frac{1}{x})$.
11. Find the value of $\int_{-1}^2 x^5 d|x|^3$.
12. If $f(x)$ is monotone on $[a,b]$, and $g(x)$ is continuous on $[a,b]$, show that $f \in R(g)$ on $[a,b]$.
13. State Rolle's theorem and mean value theorem. If $f: [0,2] \rightarrow R$ be given by $f(x) = \sqrt{2x - x^2}$, verify whether it satisfies Roll's theorem.
14. If $f_n(x) = \frac{nx}{2n+x}$ for $x \in [0, \infty)$, show that $f_n(x)$ converges uniformly on $[0, R]$ for any $R > 0$, but not on $[0, \infty)$.

(4 × 3 = 12 weightage)

(P.T.O.)

Part C: Answer any two questions. Each carries five weightage.

15. a) Every infinite bounded subset of \mathbb{R} has a limit point in \mathbb{R} .

b) If A_n is open, then $\bigcap_{n=1}^{\infty} A_n$ is open. Is this statement true? Justify your answer.

16. a) Show that every uniformly continuous function is continuous.

b) If $f(x) = \begin{cases} x & \text{if } x \text{ is rational} \\ 0 & \text{if } x \text{ is irrational} \end{cases}$ then check the continuity of $f(x)$ at $x=0$.

17. If $f \in R(\alpha_1)$ and $f \in R(\alpha_2)$ on $[a,b]$, show that $f \in R(\alpha_1 + \alpha_2)$ and

$$\int_a^b f d(\alpha_1 + \alpha_2) = \int_a^b f d(\alpha_1) + \int_a^b f d(\alpha_2).$$

18. If $f_n(x) = nx(1 - x^2)^n, x \in [0, 1]$, find the point wise limit $f(x)$ of $f_n(x)$. Verify whether $\int_0^1 f_n(x) dx \rightarrow \int_0^1 f(x) dx$.

(2 × 5 = 10 weightage)