D1AST2501 **(1 Page)** **Name.............................**
Reg.No..........................

FIRST SEMESTER M.Sc. DEGREE EXAMINATION, NOVEMBER 2025
(Regular/Improvement/Supplementary)
STATISTICS
FMST1C01-MEASURE THEORY AND INTEGRATION

Time: 3 Hours **Maximum Weightage: 30**

Part A: Answer any *four* questions. Each carries *two* weightage.

1. Define integral of a non-negative measurable function.
2. State Fatou's lemma.
3. Write short note on an integral which depends on a parameter.
4. State Holder's inequality.
5. When do you say that a sequence of measurable real valued functions converge in measure to a measurable real valued function?
6. Distinguish between Lebesgue and Lebesgue Stieltjes measure.
7. State Fubini's theorem.

(4 × 2 = 8 weightage)

Part B: Answer any *four* questions. Each carries *three* weightage.

8. What do you mean by integrable functions? If f is integrable and c is a real constant, then show that cf is integrable.
9. If f is a nonnegative function in $M(X, \aleph)$, then show that there exists a nonnegative nondecreasing sequence of simple functions $\{\varphi_n\}$ such that $f(x) = lim\varphi_n(x)$ for each $x \epsilon X$.
10. Define simple function and integral of a simple function.
11. State and prove Minkowski's inequality.
12. Distinguish between almost everywhere convergence and almost uniform convergence.
13. If λ<<μ , μ<<ϑ, then show that λ<<ϑ and $\frac{d\lambda}{d\vartheta} = \frac{d\lambda}{d\mu}\frac{d\mu}{d\vartheta}$ a.e.
14. State and prove monotone class lemma. **(4 × 3 = 12 weightage)**

Part C: Answer any *two* questions. Each carries *five* weightage.

15. State and prove monotone convergence theorem.
16. State and prove Lebesgue dominated convergence theorem.
17. State and prove Lebesgue decomposition theorem.
18. State and prove Tonelli's theorem.

(2 × 5 = 10 weightage)