D1AMT2505

Reg.No.........................

Name: ....

FIRST SEMESTER M.Sc. DEGREE EXAMINATION, NOVEMBER 2025
(Regular/Improvement/Supplementary)
MATHEMATICS
FMTH1C05: NUMBER THEORY

Time: 3 Hours **Maximum Weightage: 30**

Part A

Answer *all* questions. Each carries 1 weightage.

1. Define Euler's totient function ϕ and find $\phi(60)$.
2. Show that for all $x \geq 1$, $\sum_{n \leq x} \psi(\frac{x}{n}) = x \log x - x + O(\log x)$.
3. Show that for all $x \geq 1$, $\sum_{n \leq x} \theta(\frac{x}{n}) = x \log x + O(x)$.
4. Prove or disprove: "The Euler's totient function $\phi(n)$ is even for all $n \geq 1$". Justify.
5. Evaluate the Legendre symbol $(\frac{31}{641})$.
6. Show that the Mangoldt function Λ is not multiplicative.
7. If $\lim_{x\to\infty} \frac{\theta(x)}{x} = 1$, find $\lim_{x\to\infty} \frac{\psi(x)}{x}$. Justify.
8. Find the inverse of the matrix $\begin{pmatrix} 1 & 3 \\ 4 & 3 \end{pmatrix} \pmod{29}$.

(8 x 1 = 8 weightage)

Part B: Answer *any two* questions from *each unit.* Each carries 2 weightage.

Unit 1

9. State and prove Mobius inversion formula.
10. If f is a multiplicative function, then prove that $f^{-1}(n) = \mu(n)f(n)$ for every square free n.
11. Show that for $x \geq 2$, $\log[x]! = x \log x - x + O(\log x)$.

Unit 2

12. Show that for $x \geq 2$, $\theta(x) = \pi(x) \log x - \int_2^x \frac{\pi(t)}{t} dt$.

13. Show that $\lim_{x \longrightarrow \infty} (\frac{\psi(x)}{x} - \frac{\theta(x)}{x}) = 0$.

14. Show that there exists a constant A such that $\sum_{p \leq x} \frac{1}{p} = \log \log x + A + O(\frac{1}{\log x})$, for all $x \geq 2$, where p is a prime number.

Unit 3

15. Show that for distinct odd primes p and q, legendre symbol $(\frac{p}{q}) = -(\frac{q}{p})$ when $p \equiv q \equiv 3(mod 4)$ and $(\frac{p}{q}) = (\frac{q}{p})$ if $p \not\equiv 3(mod\ 4)$ or $q \not\equiv 3(mod\ 4)$.

16. Given that the adversary is using a 2×2 enciphering matrix A in the 26- letter alphabet. The ciphertext is "WKNCCHSSJH" and on encryption, the first word is "GIVE". Obtain the deciphering matrix and decrypt the message.

17. Determine whether 219 is a quadratic residue modulo 383.

(6 x 2 = 12 weightage)

Part C: Answer *any two* questions. Each carries 5 weightage.

18. State and prove Euler's summation formula.

19. State and prove Abel's identity.

20. State and prove Gauss lemma.

21. Show that the partial sums of the series $\sum_{n=1}^{\infty} \frac{\mu(n)}{n}$ is bounded.

(2 x 5= 10 weightage)