

**FIRST SEMESTER M. Sc. DEGREE EXAMINATION, NOVEMBER 2025**  
(Regular/Improvement/Supplementary)

**MATHEMATICS**  
**FMTH1C04- DISCRETE MATHEMATICS**

**Time: 3 Hours**

**Maximum Weightage: 30**

**Part A: Answer *all* questions. Each carries 1 weightage.**

1. Define a total order on a set  $X$ . Give examples.
2. Write the following Boolean function in its disjunctive normal form.  
$$f(x_1, x_2, x_3) = (x_1 + x_2')x_3' + x_2x_1'(x_2 + x_1'x_3)$$
3. Prove or disprove: In any graph  $G$ , the number of vertices of odd degrees is even.
4. Check whether the sequence (7, 6, 5, 4, 3, 3, 2) is graphical or not.
5. If  $G$  is a simple planar graph with at least 3 vertices, then show that  $m \leq 3n - 6$ , where  $m$  and  $n$  are number of edges and vertices respectively.
6. Define the Cartesian product of two graphs  $G_1$  and  $G_2$ . Give example.
7. Define a grammar  $G$ .
8. Prove that  $(\omega^R)^R = \omega$  for all  $\omega \in \Sigma^*$ .

**(8 × 1 = 8 weightage)**

**Part B: Answer any *two* questions from each unit. Each carries 2 weightage.**

**Unit 1**

9. Let  $X$  be a finite set and  $\leq$  a partial order on  $X$ . Define a binary relation  $R$  on  $X$  by  $xRy$  if and only if  $y$  covers  $x$  (with respect to  $\leq$ ). Then prove that  $\leq$  is generated by  $R$  i.e., the smallest order relation on  $X$  containing  $R$ .
10. Let  $(X, +, \cdot, ', 0, 1)$  be a finite Boolean algebra. Then prove that:
  - (i) Every nonzero element of  $X$  contains at least one atom.
  - (ii) Every two distinct atoms of  $X$  are mutually disjoint.

**(P.T.O.)**

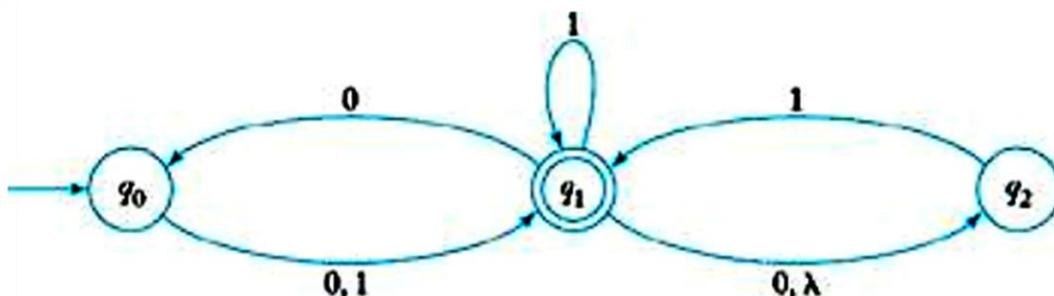
11. If  $(X, +, \cdot, ')$  is a Boolean algebra, prove that for all  $x, y, z$  in  $X$ ,  
 (i)  $(x')' = x$             (ii)  $x + x = x$ .

### Unit 2

12. (i) Define a connected graph.  
 (ii) Among the two graphs  $G$  and  $G^c$ , atleast one of them is connected. Justify.
13. (i) For a connected plane graph  $G$ ,  $n - m + l = 2$ , where  $l$  denotes the number of faces of  $G$ .  
 (ii) For any simple planar graph  $G$ ,  $\delta(G) \leq 5$ .
14. Prove that a connected graph  $G$  with at least two vertices contains at least two vertices that are not cut vertices.

### Unit 3

15. Find a dfa that accepts all the strings on  $\{0,1\}$ , except that containing the substring 001.
16. Which of the given strings are accepted by the following nfa.  
 00, 01001, 10010, 000, 0000.



17. Let  $M = (Q, \Sigma, \delta, q_0, F)$  be a dfa and let  $G_M$  be its associated transition graph. Then for every  $q_i, q_j \in Q$  and  $w \in \Sigma^+$ ,  $\delta^*(q_i, w) = q_j$  if and only if there is in  $G_M$  a walk with label  $w$  from  $q_i$  to  $q_j$ .

(6 × 2 = 12 weightage)

### Part C: Answer any two questions. Each carries 5 weightage.

18. Let  $(X, \leq)$  be a poset and  $A$  a nonempty finite subset of  $X$ . Then prove that  $A$  has atleast one maximal element. Also prove that  $A$  has a maximum element if and only if  $A$  has a unique maximal element.

19. (i) Prove that an edge  $e = xy$  of a graph  $G$  is a cut edge of a connected graph  $G$  if and only if  $e$  does not belong to any cycle of  $G$ .
- (ii) Prove that an edge  $e = xy$  of a graph  $G$  is a cut edge of a connected graph  $G$  if and only if there exist vertices  $u$  and  $v$  such that  $e$  does not belong to every  $u - v$  path in  $G$ .
20. (i) Prove that the number of edges in a tree with  $n$  vertices is  $n - 1$ . Is the converse true? Justify.
- (ii) State and prove the characterization of trees involving cut edges.
21. Let  $L$  be the language accepted by a nondeterministic finite acceptor  $M_N = (Q_N, \Sigma, \delta_N, q_0, F_N)$ . Then prove that there exist a dfa  $M_D = (Q_D, \Sigma, \delta_D, \{q_0\}, F_D)$  such that  $L = L(M_D)$ .

**(2 × 5 = 10 weightage)**