D1AST2405 (2 Pages) Name...........................
Reg.No..........................

FIRST SEMESTER M.Sc. DEGREE EXAMINATION, NOVEMBER 2024
(Regular/Improvement/Supplementary)

STATISTICS
DISTRIBUTION THEORY

Time: 3 Hours **Maximum Weightage: 30**

Part A: Answer any *four* questions. Each carries *two* weightage.

1. Define probability generating function. Examine whether $P(t) = \frac{2}{1+t}$ can be a probability generating function.
2. Define power series family of distributions. Identify two members of the family.
3. If X is distributed according to U(0,1), show that -2logX is distributed according to Chi-square.
4. Define standard Laplace distribution. Obtain its characteristic function.
5. If *(X, Y)* has a bivariate normal distribution, show that *X* and *Y* are independent if and only if $\rho = 0$.
6. Explain the concept of mixture distribution.
7. Define F statistic and write its probability density function.

(4 × 2 = 8 weightage)

Part B: Answer any *four* questions. Each carries *three* weightage.

8. If X1, X2, ..., Xn are k independent Poisson variates with parameters $\lambda_1, \lambda_2, \lambda_3, ...,,,, \lambda_k$, find the conditional distribution of $p(X_1 \cap X_2 \cap ...... \cap X_n / X)$ where $X = X_1 + X_2 + ... + X_k$.
9. If X is a non-negative integer valued random variable, then find the PGF of:
   (i) $P(X \leq n)$ (ii) $P(X = 2n)$.
10. Derive the characteristic function of a Weibull distribution.
11. Let X and Y be independent Gamma variates $G(\alpha_1, \beta)$ and $G(\alpha_2, \beta)$ respectively. Examine whether $X + Y$ and $\frac{X}{Y}$ are independent.
12. Define Pearson system of distributions. Examine whether the beta distribution belongs to the family.
13. Define order statistics. Find the distribution of sample median for a sample of size 5 from a population having the distribution function F(x) = 1 - e^{-x}, $0 < x < \infty$.
14. Define central t statistic. Write the probability distribution of t and drive its moments.

(4 × 3 = 12 weightage)

(P.T.O.)

Part C: Answer any *two* questions. Each carries *five* weightage.

15. Obtain the expression for the moment generating function of a power series distribution. Also obtain the expression for the recurrence relation for cumulants of power series distribution. Show that binomial and Poisson distributions belong to the power series family.
16. Define Weibull distribution. Let X_i , I = 1, 2,...., n are i.i.d random variable. Show that Min $(X_1, X_2, \dots \dots\dots, X_n,)$ has a Weibull distribution if and only if each X_i, has a Weibull distribution.
17. If X_1 and X_2 are independent Chi-square variates with n_1 and n_2 d.f respectively, then show that U = $\frac{X1}{X1+X2}$ and V = X_1 + X_2 are independently distributed, U as $\beta_1(\frac{n_1}{2}, \frac{n_2}{2})$ variate and V as a chi- square variate with $(n_1 + n_2)$ d.f.
18. Derive the distributions of sample mean and the sample variance of a sample from a normal population and show that they are independently distributed.

(2 × 5 = 10 weightage)