D1AST2403 (2 pages) Name.............................

Reg.No..........................

FIRST SEMESTER M.Sc. DEGREE EXAMINATION, NOVEMBER 2024

(Regular/Improvement/Supplementary)

STATISTICS

FMST1C03-ANALYTICAL TOOL FOR STATISTICS II

Time: 3 Hours **Maximum Weightage: 30**

Part A: Answer any *four* questions. Each carries *two* weightage.

1. Define subspace of a vector space. State and prove a necessary and sufficient condition for a subset of vector space to be a subspace.
2. Show that an orthogonal set of non-null vectors is linearly independent.
3. Define generalized inverse of a matrix. Show that it is not unique.
4. If A and B are matrices of the same order, then show that $rank(A+B) \leq rank(A) + rank(B)$.
5. Define a skew Hermitian matrix. Discuss the nature of eigen values of a skew Hermitian matrix.
6. Discuss the algebraic and geometric multiplicities of an eigen value.
7. Define a quadratic form. Explain the classification of quadratic forms.

(4 × 2 = 8 weightage)

Part B: Answer any *four* questions. Each carries *three* weightage.

8. Show that a set of non-zero vectors is linearly dependent if and only if a member in the set is a linear combination of the preceding vectors.
9. If S and T are subspaces of a vector space, then show that $S \cap T$ and $S+T$ are also subspaces.
10. Discuss the method of computing the inverse of a matrix using partitioned method.
11. Show that any set of characteristic vectors corresponding respectively to a set of distinct characteristic roots of a matrix is linearly independent.
12. Explain the spectral decomposition of a Hermitian matrix. Discuss its properties.
13. Define Moore-Penrose inverse. Show that it is unique.
14. If A is a real symmetric matrix with characteristic roots $\lambda_1 < \lambda_2 < ... < \lambda_n$, then show that $Sup \dfrac{X'AX}{X'X} = \lambda_n$ and $Inf \dfrac{X'AX}{X'X} = \lambda_1$.

(4 × 3 = 12 weightage)

(P.T.O.)

Part C: Answer any *two* questions. Each carries *five* weightage.

15. Define an orthonormal basis of a subspace. Explain Gram-Schmidt orthogonalization process using an example.

16. If A and B are two square matrices of order n, then show that

$$Max\{\upsilon(A),\upsilon(B)\}\leq\upsilon(AB)\leq\upsilon(A)+\upsilon(B),$$

where $\upsilon(A),\upsilon(B)$, and $\upsilon(AB)$ are the nullities of A, B, and AB respectively.

17. Identify the nature of eigen values of a Hermitian matrix. Show that the eigen vectors corresponding to distinct eigen values of a Hermitian matrix are orthogonal.

18. State and prove the necessary and sufficient condition for the positive definiteness of a real quadratic form. Examine whether the quadratic form $x^2-2xy+2y^2$ is positive definite.

(2 × 5 = 10 weightage)